# Transcomputation - Answers 1

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## 1 Evaluate the following transreal expressions

1.1 $\infty + 1 = \frac{1}{0} + \frac{1}{1} = \frac{1\times1\ +\ 1\times0}{0\times1} = \frac{1}{0} = \infty$

1.2 $\infty - 1 = \frac{1}{0} - \frac{1}{1} = \frac{1}{0} + \frac{-1}{1} = \frac{1\times1\ +\ (-1)\times0}{0\times1} = \frac{1}{0} = \infty$

1.3 $\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{1+1}{0} = \frac{2}{0} = \frac{1}{0} = \infty$

1.4 $\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{1}{0} + \frac{-1}{0} = \frac{1-1}{0} = \frac{0}{0} = \Phi$

1.5 $3 \times \infty = \frac{3}{1} \times \frac{1}{0} = \frac{3\times1}{1\times0} = \frac{3}{0} = \frac{1}{0} = \infty$

1.6 $3 \div \infty = \frac{3}{1} \div \frac{1}{0} = \frac{3}{1} \times \frac{0}{1} = \frac{3\times0}{1\times1} = \frac{0}{1} = 0$

1.7 $\infty \div (-3) = \frac{1}{0} \div \frac{-3}{1} = \frac{1}{0} \times \frac{1}{-3} = \frac{1}{0} \times \frac{-1}{3} = \frac{1\times(-1)}{0\times3} = \frac{-1}{0} = -\infty$

1.8 $\infty \div \infty = \frac{1}{0} \div \frac{1}{0} = \frac{1}{0} \times \frac{0}{1} = \frac{1\times0}{0\times1} = \frac{0}{0} = \Phi$

## 2 Check the transreal, distributivity rules.

We cannot check $a(b + c) = ab + ac$ for all transreal $a$, $b$, $c$ so, instead, we check at critical values. For example, we may use these eight ordered values: $-\infty < -R < r < 0 < r < R < \infty$ and $\Phi$. Here $R$ and $r$ are arbitrarily chosen, but fixed, values; hence they cover the full range of real numbers. Testing all combinations of $a$, $b$, $c$ involves choosing each one of 8 values for $a$ and constructing an $8 \times 8$ multiplication table for it, with rows for each of the eight $b$ and columns for each of the eight $c$. Thus we evaluate $a(b = c) = ab + ac$ a total of $8^3 = 512$ times. Stop when you feel you have had enough practice! Better still, stop when you have checked all these cases in a machine algebra system. In either case, make your life easy and use symmetries to reduce the amount of work.

## 3 Prove $0^0 = \Phi$

Hint: $e^{-\infty} = 0$, $e^{\infty} = \infty$, $e^{\Phi} = \Phi$. The transreal, natural logarithm, $\ln y$, is the inverse of the transreal exponential, $e^x$.

Proof: $0^0 = e^{\ln 0^0} = e^{0 \ln 0} = e^{0 \times (-\infty)} = e^{\Phi} = \Phi$.

## 4 Prove $e^{-\infty} = 0$, $e^{\infty} = \infty$, $e^{\Phi} = \Phi$

Keep track of your assumptions and reflect on how you would develop transreal analysis.

This is an open-ended task. Search the web to see how other people are working on problems like this. Explore your own solutions and compare them to the very simple approach in the next exercise sheet.