Transmathematics and how to execute a program in less than one clock cycle

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Agenda

- What is transmathematics?
- Acceptance of the complex numbers
- Acceptance of the transreal numbers
- Execute a program in less than one clock cycle

Transmathematics

- Deals with total systems that have no exceptions
- Dividing by zero is not an exception
- Solves physical problems at singularities
- Allows exception-free programming
- Enables unbreakable pipelines

Transmathematics

- Transmathematica journal
- Transmathematica conference
- Totallica company

Complex Numbers

- It used to be impossible to find the square root of a negative number
- $\sqrt{-4} = ?$

Complex Numbers

- Define $i = j = \sqrt{-1}$

Complex Numbers

- Now $(i2)^2 = i^2 2^2 = -1 \times 4 = -4$
- So $\sqrt{-4} = i2$
- But people argued about this for 400 years!

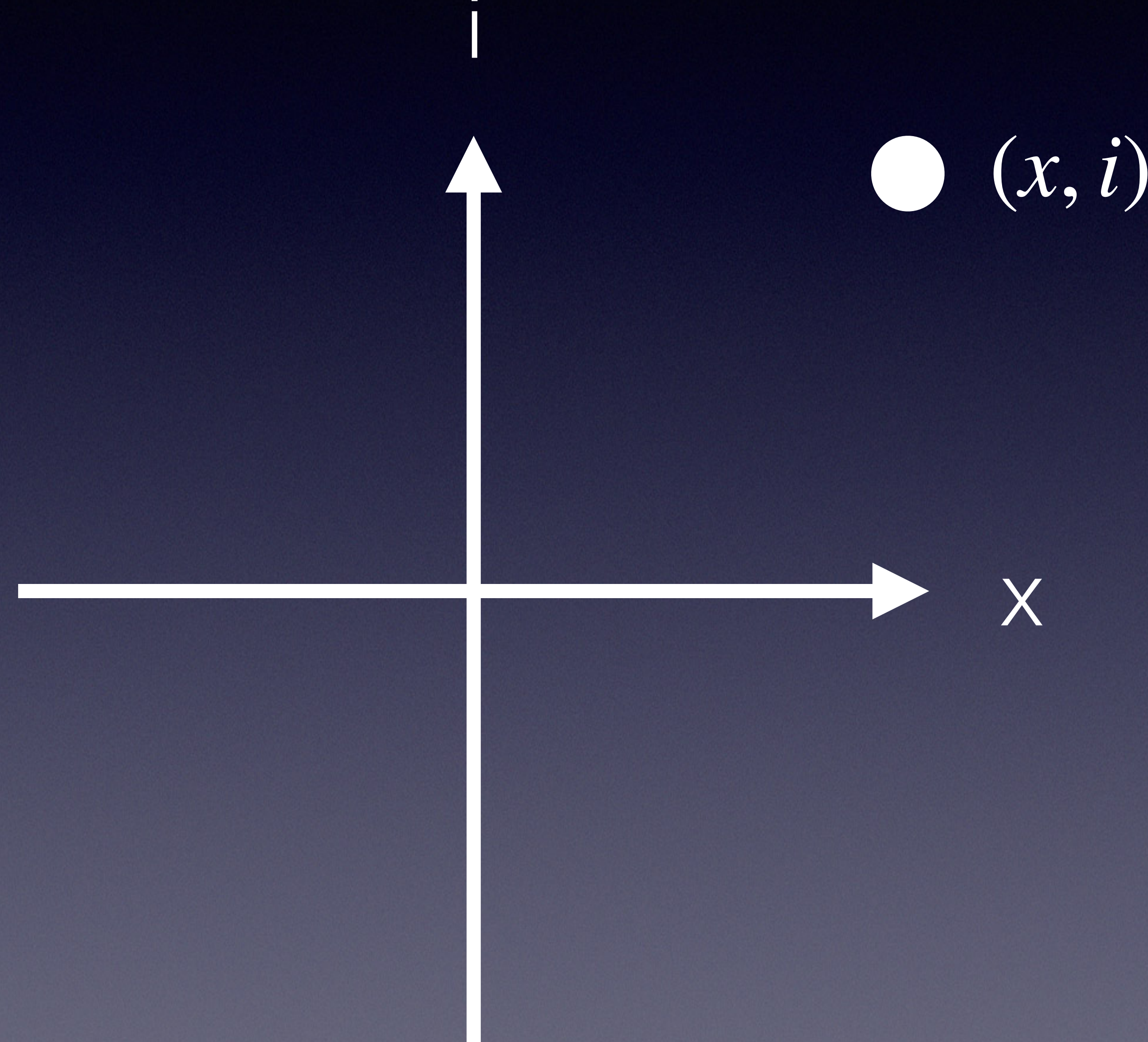
Complex Acceptance

- Complex numbers were constructed from real numbers
- So complex arithmetic is consistent if real arithmetic is
- So complex arithmetic cannot be disproved in its own terms, only real arithmetic can be disproved

Complex Acceptance

- Controversy over complex numbers ended when they were given a geometrical interpretation

Complex Geometry



Complex Acceptance

- Fully accepted when useful applications were developed

Complex Acceptance

- Construction from the real numbers
- Geometrical models
- Today, machine proofs of consistency
- Today, many useful applications

Transreal Numbers

Transreal numbers, t, are proper fractions of real numbers, with a non-negative denominator, d, and a numerator, n, that is one of -1, 0, 1 when d = 0

$$t = \frac{n}{d}$$

With k a positive constant:

$$-\infty = \frac{-k}{0} = \frac{-1}{0} \qquad \Phi = \frac{0}{0} \qquad \infty = \frac{k}{0} = \frac{1}{0}$$

Negative Denominators

An improper fraction may have a negative denominator (-k) which must be made positive *before* any transarithmetical operator is applied

$$\frac{n}{-k} = \frac{-n}{-(-k)} = \frac{-1 \times n}{-1 \times (-k)} = \frac{-n}{k}$$

Multiplication

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Division

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

Addition of Two Infinities

$$\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{1+1}{0} = \frac{2}{0} = \frac{1}{0} = \infty$$

$$\infty + (-\infty) = \frac{1}{0} + \frac{-1}{0} = \frac{1-1}{0} = \frac{0}{0} = \Phi$$

$$-\infty + \infty = \frac{-1}{0} + \frac{1}{0} = \frac{-1+1}{0} = \frac{0}{0} = \Phi$$

$$-\infty + (-\infty) = \frac{-1}{0} + \frac{-1}{0} = \frac{-1+(-1)}{0} = \frac{-2}{0} = \frac{-1}{0} = -\infty$$

General Addition

$$\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$$

Subtraction

$$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

Associativity

$$a+(b+c)=(a+b)+c$$

$$a\times(b\times c)=(a\times b)\times c$$

Commutativity

$$a + b = b + a$$

$$a \times b = b \times a$$

Partial Distributivity

$$a(b+c)=ab+ac$$

When $a \neq \pm\infty$ or

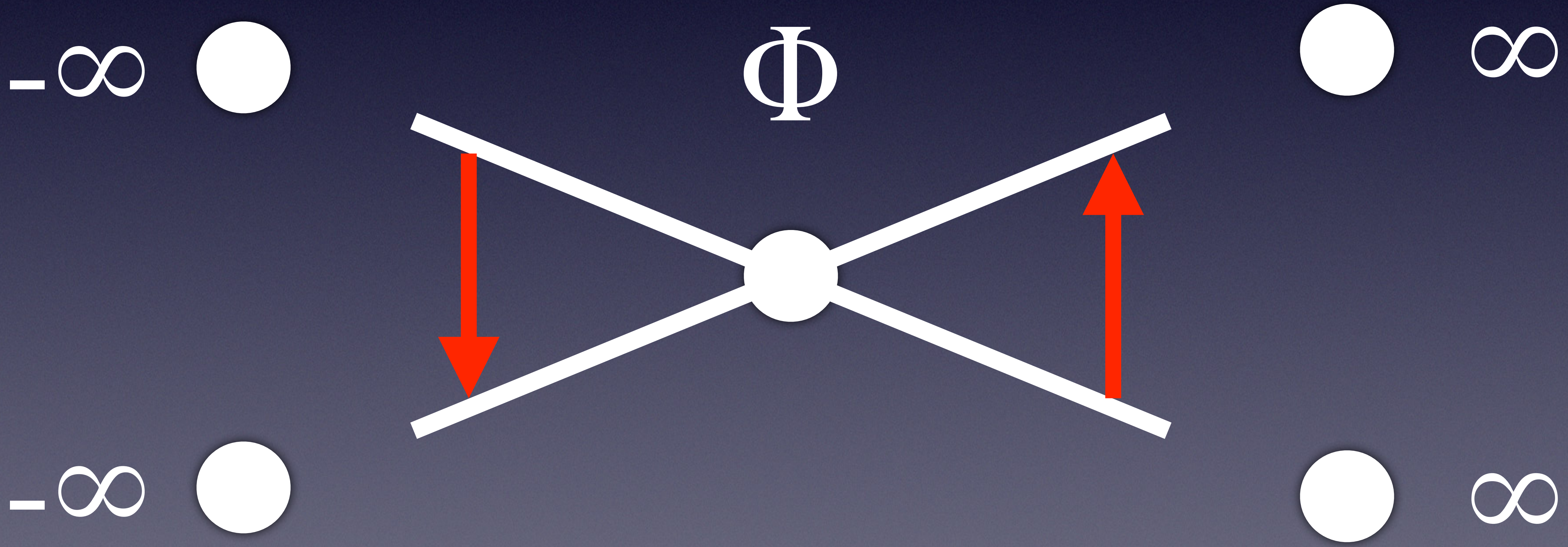
$bc > 0$ or

$$(b+c)/0=\Phi$$

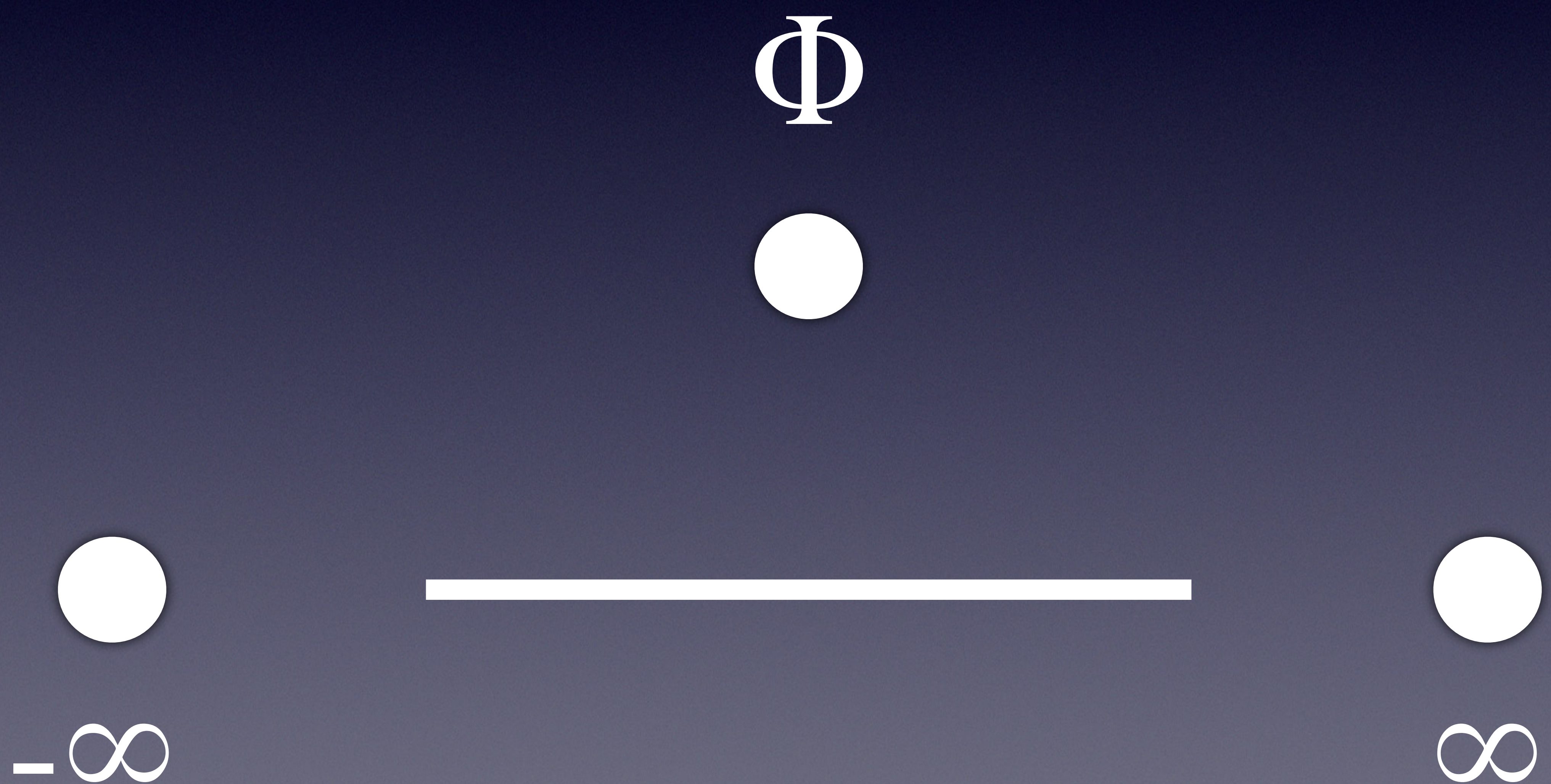
Transreal Acceptance

- Transreal arithmetic proved consistent by machine proof (2006)
- Transreal (2016) and transcomplex (2014) arithmetic proved consistent by construction from, respectively, the real and complex numbers
- So transreal and transcomplex arithmetic are consistent if real arithmetic is and can only be disproved if real arithmetic is

Projective Geometry

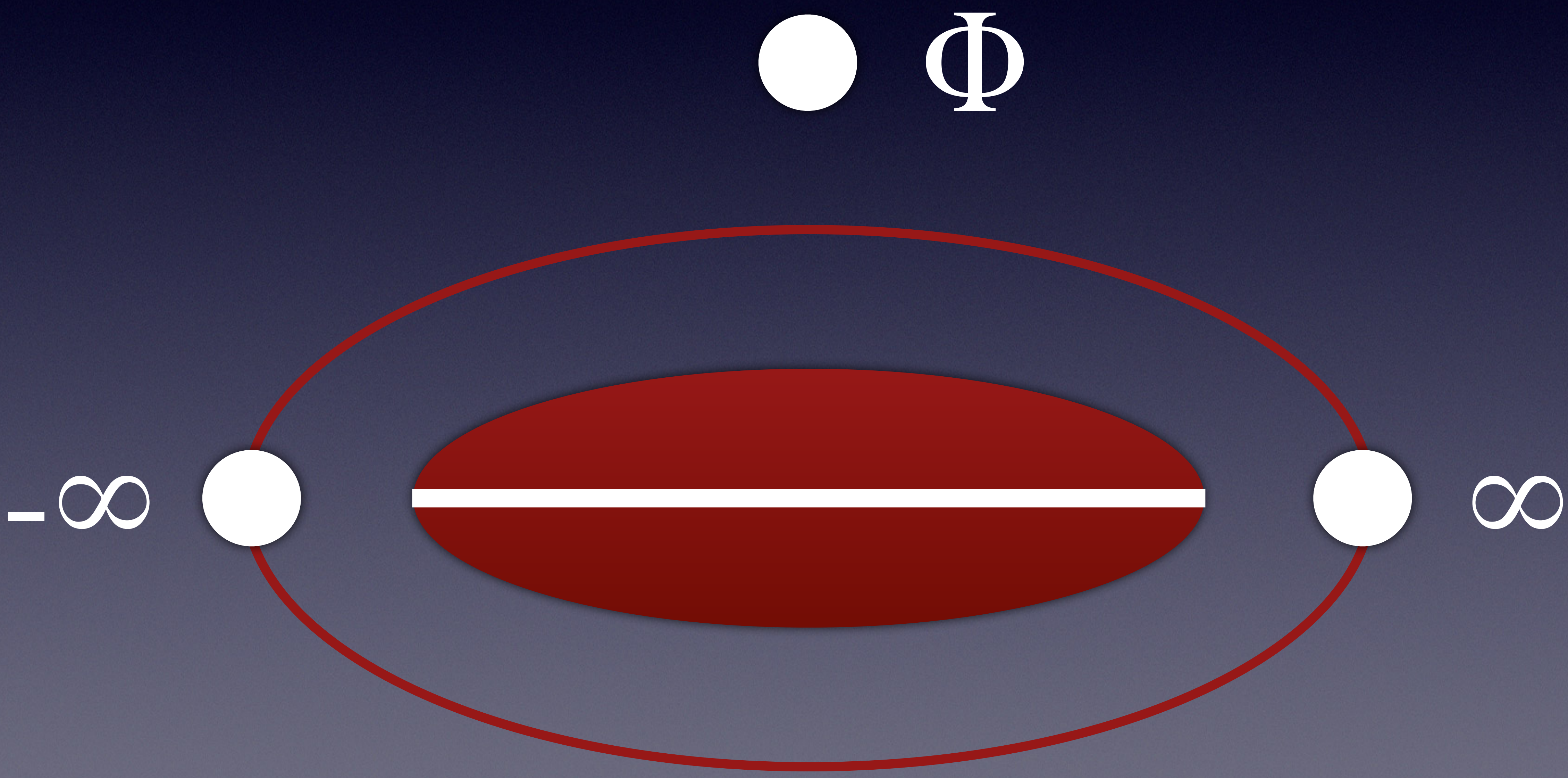


Transreal-Number Line



Transcomplex Plane

Revolution of the transreal number line



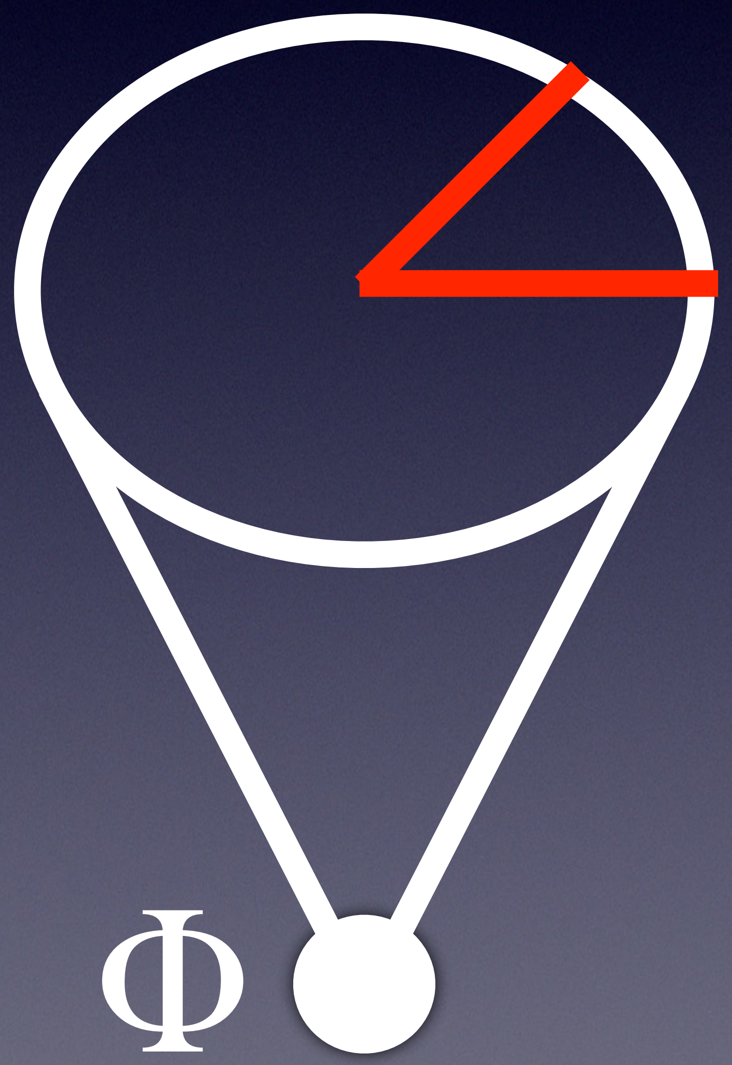
Transangle

Real and nullity angles are arc length divided by radius

But where are the infinity angles?

Transangle

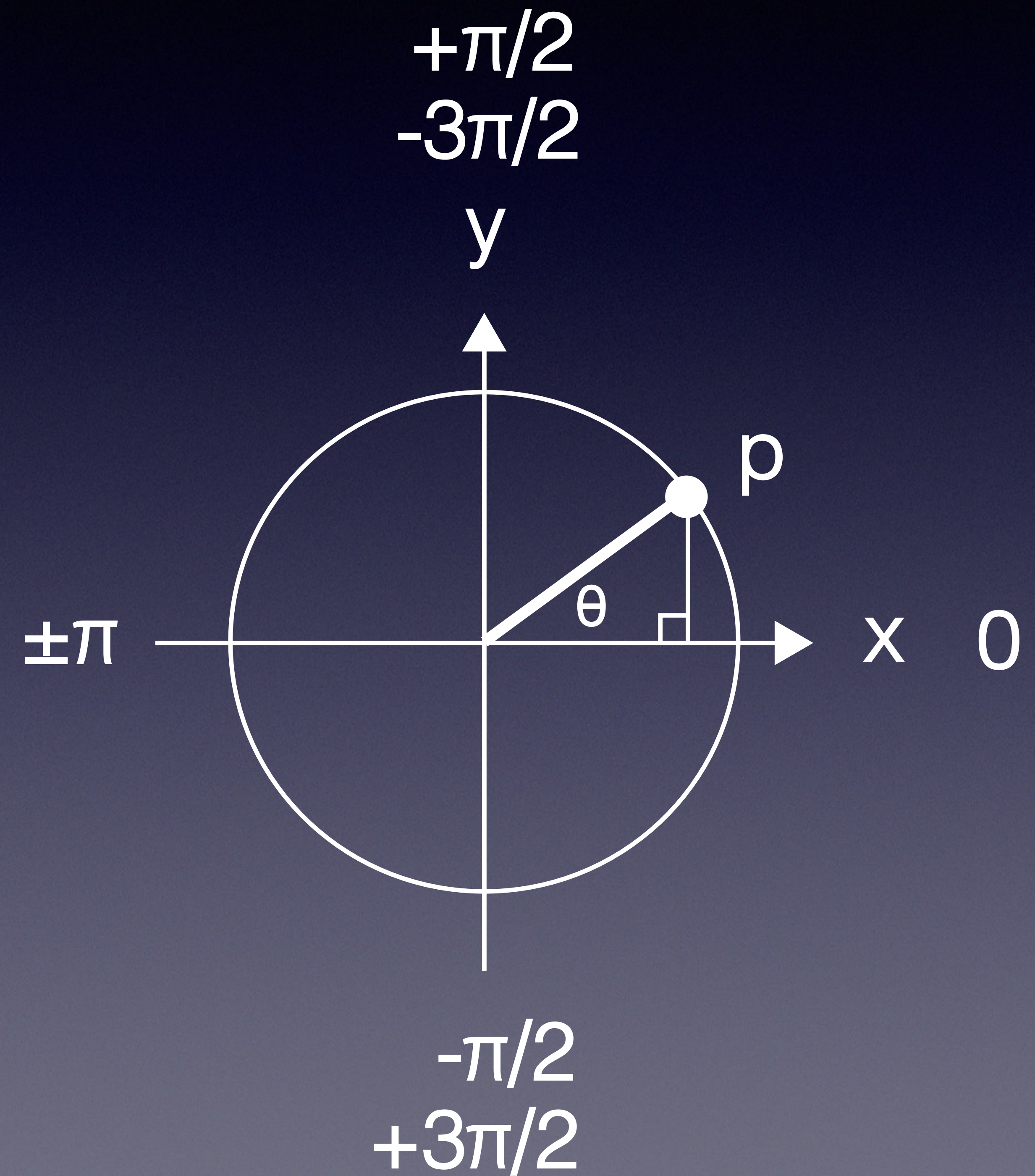
The infinity angle is the winding at the apex of the unit cone



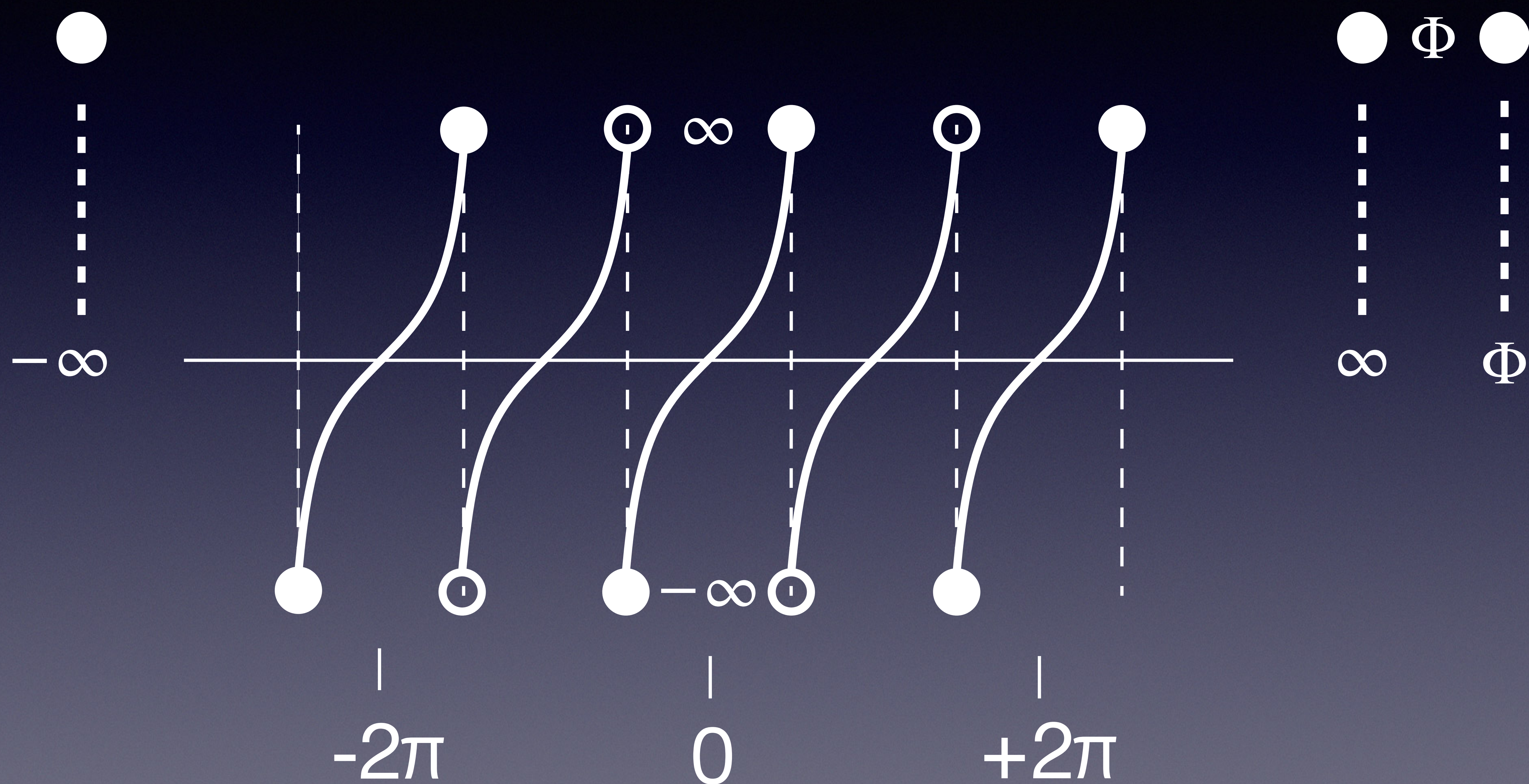
Transreal Acceptance

- Transreals have a geometrical model
- Transcomplexes have a geometrical model
- Transreal angles have a geometrical model

Transtangent



Transtangent



Transtangent

- Is defined for all transreal angles
- Is single valued everywhere
- The real values of the transtangent have period π
- The infinite values of the transtangent have period 2π
- The nullity value of the transtangent occurs at $-\infty$, ∞, Φ

Transtangent

- Transtangent definition on a triangle is consistent with
- Transtangent definition as a power series

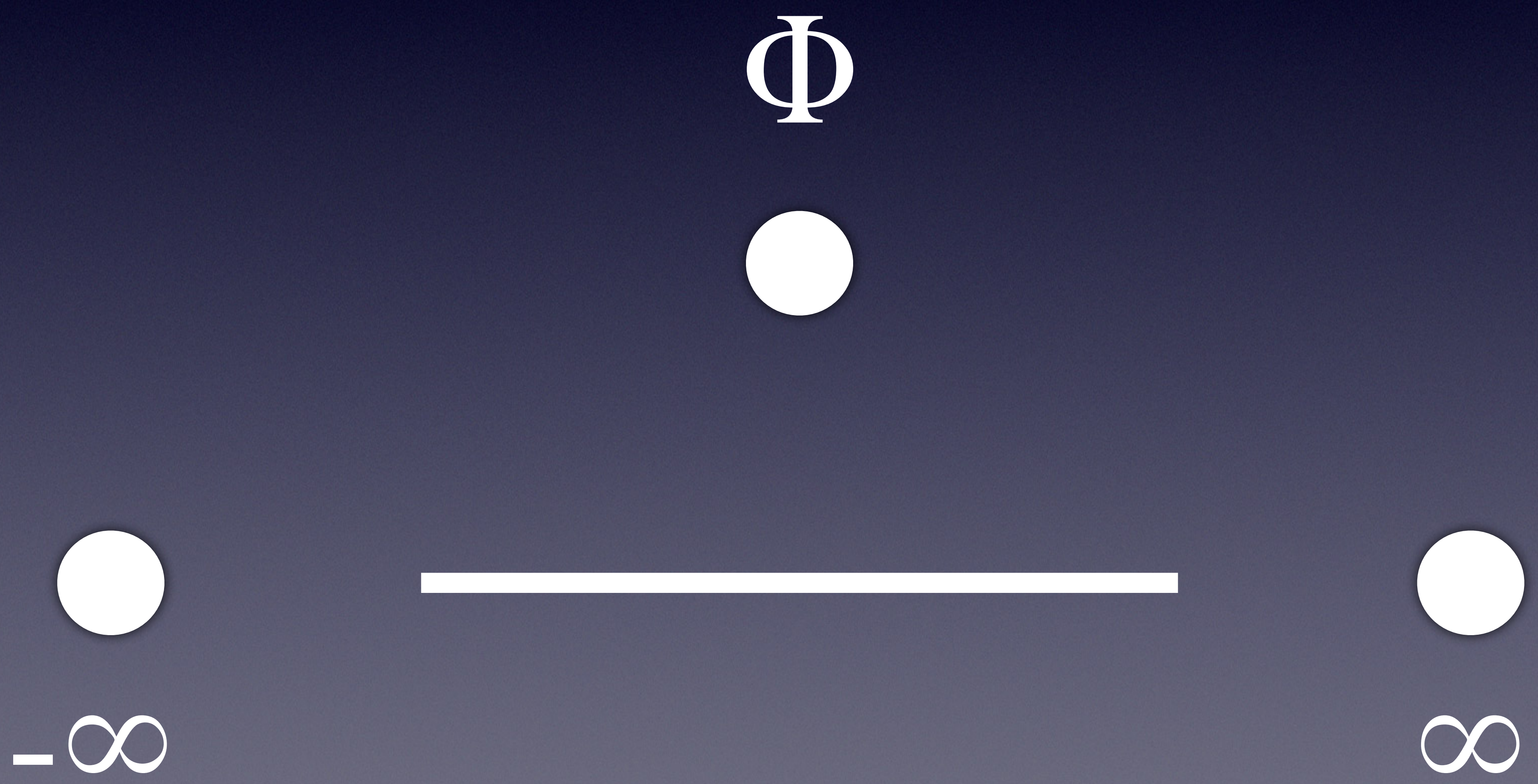
Transmathematics

- Transalgebra including Trans-Boolean Algebra
- Transreal & transcomplex elementary functions
- Transreal and transcomplex calculus
- Transvectors

Transmathematics

- Transreal arithmetic is so general it is beginning to solve problems in logic
- Transreal calculus is so general that it is beginning to solve problems in real calculus and mathematical physics (no empirical verification - yet?!)

Transreal-Number Line



Nullity Force

- There is no component of nullity on the extended-real-number line so nullity forces have no, that is zero, effect on the extended-real universe where we live

Newton's Law 1

- A mass is accelerated only by a positive or negative force, not a zero or nullity force

Newton's Law 2

- $F = ma$ when $0 < m < \infty$ and a is transreal
- $a = F / m$ when $0 < m < \infty$ and F is transreal
- $m = F / a$ when a, F are transreal. When the computed mass is real, it is determined. When the computed mass is nullity, the true, finite mass, is hidden (but can be discovered by its gravitational effects)

Newton’s Law 3

- To any action, F, there is always an opposite and equal reaction, -F

Information

- Real numbers have more information than infinite numbers because both the sign and magnitude of real numbers is non-absorptive but the magnitude of infinite numbers is absorptive

- Infinite numbers have more information than nullity because nullity's sign and magnitude is absorptive

- Hypothesis: physical systems always adopt the transreal configuration with the highest possible information

Black Hole

- Suppose we have two, same charged, massive, point particles at the singularity of a black hole

- Attraction $F_g = G\dfrac{m_1 m_2}{r^2} = G\dfrac{m_1 m_2}{r^2} = \infty$

- Repulsion $F_e = k_e \dfrac{q_1 q_2}{r^2} = k_e \dfrac{q_1 q_2}{r^2} = -\infty$

- Nett force $F = F_g + F_e = \infty - \infty = \Phi$

Black Hole

- The particles are bound by a nullity force at the singularity so are free to move but are not compelled to move
- A quantal fluctuation in position may move some effective mass away from the singularity - if it inflates, it may leave the event horizon, if not it falls back into the singularity in a convection current

Black Hole

- The convection current perturbs the event horizon:

- Evaporation is faster than predicted by Hawking because a bumpy and roiling event horizon has a larger surface area than a spherical one

- Heating outside the event horizon is non-monotonic because a bumpy and roiling event horizon accelerates local particles differentially

Electronics

- For the last one hundred years, electronic circuits have been known with properties 0/0
- The circuits are measurable
- There is no theory of what to do with them

Serial Computation

- Maximises latency to first and successive solutions
- Efficiency decreases with increasing ratio of processor to memory speed because of memory bottleneck
- Efficiency decreases with increasing size

Parallel Computation

- Minimises latency to first solution
- Efficiency decreases with increasing ratio of processor to memory speed
- Efficiency decreases with increasing size
- An infinitely fast or big parallel von Neuman machine has efficiency zero and does zero work

Dataflow

- Maximises concurrency and throughput
- Processors run at memory speed
- Constant efficiency

Serial Processing

Instruction 1

Data 1

Instruction 2

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2 Data 1

Instruction 3

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3 Data 1

Instruction n

Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data 1

Serial Processing

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Serial Processing

Instruction 1

Instruction 2

Instruction 3

Instruction n

Data n

Slipstream Computer

- Dataflow means travel time is proportional to distance so never stalls on memory bottleneck
- Dataflow means I/O bandwidth is independent of the number of mills
- Totallity means that if a program compiles it has no logical exceptions so it can crash only on a physical fault
- Totallity means pipelines never break

Slipstream Processing

Instruction 1

Data 1

Instruction 2

Instruction 3

Instruction n

Slipstream Processing

Instruction 1 Data 2

Instruction 2 Data 1

Instruction 3

Instruction n

Slipstream Processing

Instruction 1 Data 3

Instruction 2 Data 2

Instruction 3 Data 1

Instruction n

Slipstream Processing

Instruction 1	Data n
Instruction 2	Data 3
Instruction 3	Data 2
Instruction n	Data 1

Slipstream Processing

Instruction 1	
Instruction 2	Data n
Instruction 3	Data 3
Instruction n	Data 2

Slipstream Processing

Instruction 1

Instruction 2

Instruction 3 Data n

Instruction n Data 3

Slipstream Processing

Instruction 1

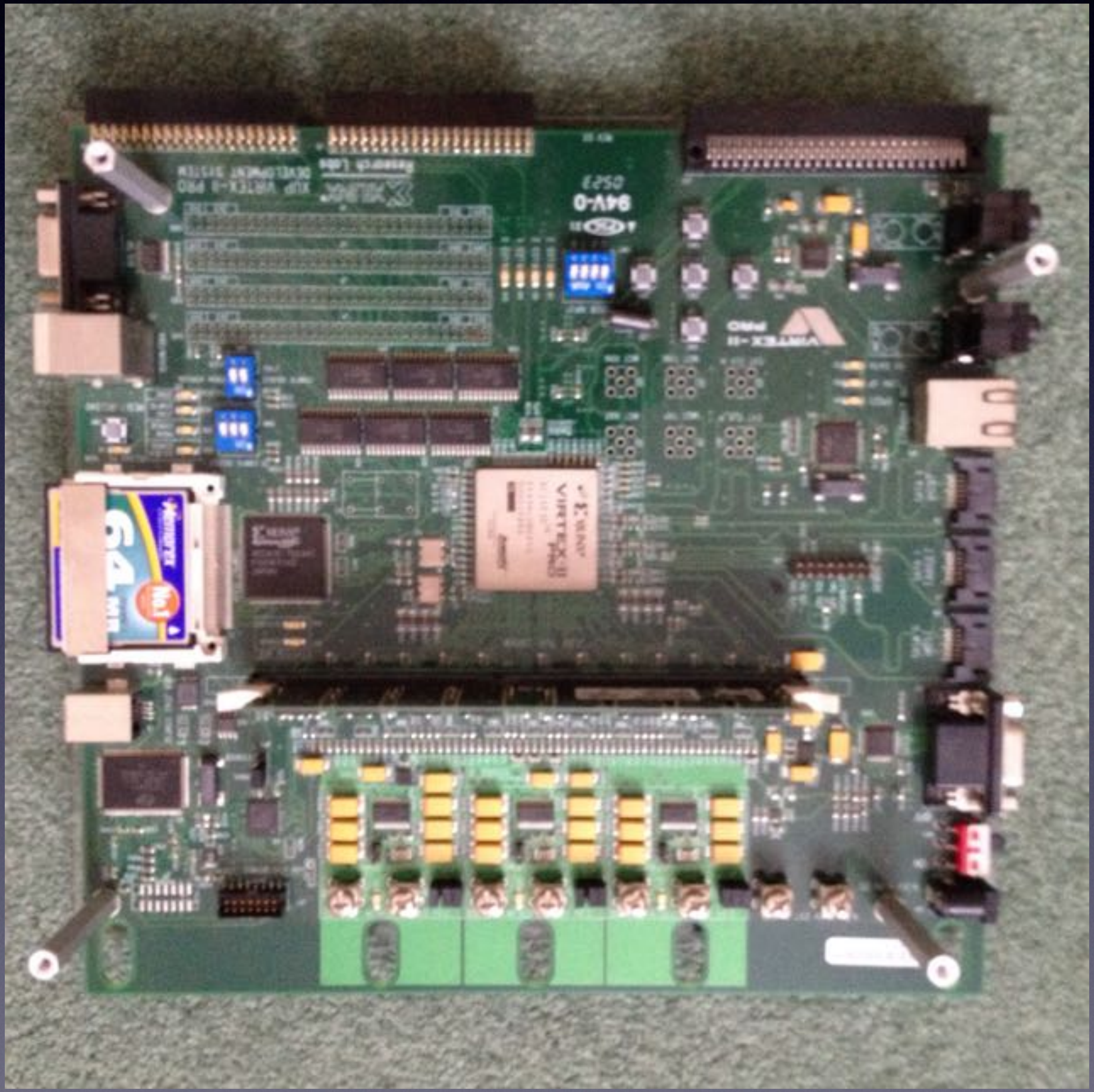
Instruction 2

Instruction 3

Instruction n

Data n

FPGA Prototype



Architectural Prototype

- Token = 12-bit header + 80-bit float datum
- 64 k mills per chip
- 2 M mills per board
- 16 M mills per cabinet
- 20 kW per unweighted Wassenaar Peta FLOP (PWFLOP)

I/O

- Systolic arrays have one dimensional I/O which has linear scaling and is impossible to fabricate
- Architectural prototype uses zero dimensional I/O which has constant (no) scaling and can be fabricated

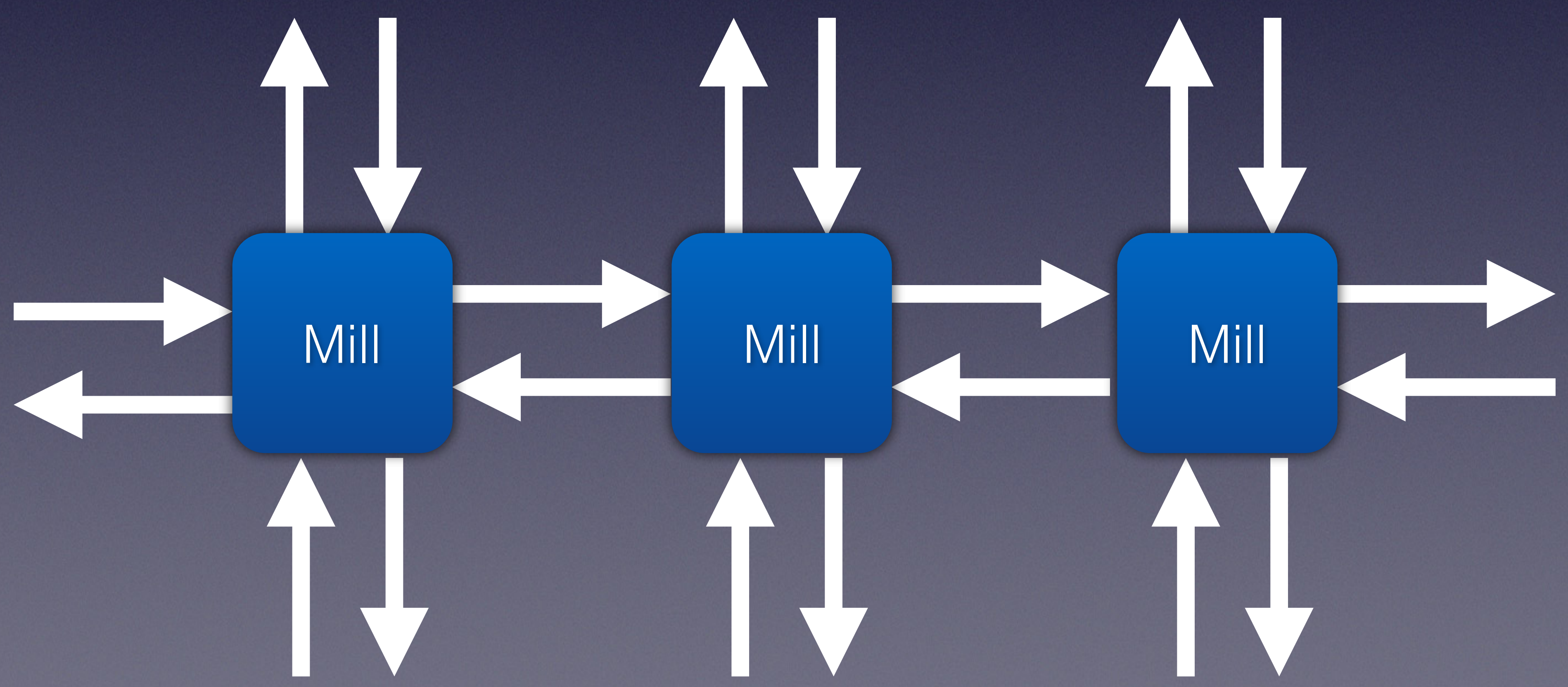
Relative Addressing

- Fixed size, relative address implements an address horizon in an arbitrarily large machine and maintains constant computational efficiency regardless of the size of the machine

- Small horizon keeps the token header small

Processor Grid

- Square grid of mills
- Pipelined communication not just nearest neighbour



Slipstream

- A grid of mills may be arranged in any dimensionality of space (2D is convenient for chips!)

- The nodes of the grid are coloured by the configuration state of the mills

- A Turing program is a directed graph in a grid

- A slipstream program is an acyclic graph in a grid

Slipstream

- Slipstream programs execute in a cadence (period) of the longer of the input and output times

- Programs with shared data, such as molecular dynamics, may have many copies of a program that share data so the average cadence is less than one and the limit of the cadence, with increasing machine size, can be zero!

Slipstream

- A practical slipstream machine cannot achieve a cadence of zero
- But the ratio of the execution time of a practical slipstream machine versus a practical von Neumann serial or parallel machine can be infinity

Transreal Acceptance

- Construction from the real numbers
- Geometrical models
- Today, machine proofs of consistency
- Tomorrow, many useful applications