Wallace's Letter

WALLACE Defining the ratio $0/0 = \Phi$ as the number *nullity* is quite interesting, but can nullity be interpreted as a probability?

JAMES Wow! I don't know. If it can't, I'm in trouble. Let's see. How is probability defined empirically?

WALLACE Empirically? Well, one carries out an experiment, such as tossing a coin, lots of times. Let's say we repeat the coin toss r times. If the coin comes down heads we record the number of heads as the *outcome,* h, if it comes down tails, we record the number of tails as the outcome, t. Then the *frequency* of a head is the ratio h/r and the frequency of a tail is t/r. We assume that the frequency tends to the probability over sufficient trials. In the limit, we say that the probability of a head is h/r and the probability of a tail is t/r.

JAMES Suppose you carry out zero repetitions of the coin toss. What are the outcomes?

WALLACE Well, if I don't toss the coin the number of heads is zero and the number of tails is zero.

JAMES So what are the frequencies?

WALLACE It's obvious! The frequency of a head is $h/r = 0/0 = \Phi$. Oh. I see. The frequency of a head is nullity! And the frequency of a tail is nullity! So their probabilities are nullity! Why did I even have to ask?

JAMES Because, my dear friend, it is not as simple as that. Are there any mathematical constraints that hold between the outcomes of an experiment?

WALLACE Sure. If the outcomes are independent, as they would be with a coin, then $h + t = r$. So if we don't toss the coin we have $h = t = r = 0$ and our formula, $h + t = r$, holds.

JAMES Can you toss a coin an infinite number of times?

WALLACE In a practical sense? No. Theoretically, yes.

JAMES Let's take the theoretical case. If I have a fair coin and toss it ten times, I would expect the frequency of a head to be about one half. Let's say that in the fair case the frequency of a head is $f = 1/2$ and that in an unfair case it is $u = 9/10$. If I repeat the experiment lots of times then the ratios will come out about the same. But if I carry out the experiment infinitely often and record the outcomes then $h = t = r = \infty$. The formula $h + t = r$ still holds, but now the frequency of a head is $f = h/r = \infty/\infty = \Phi$ and $u = h/r = \infty/\infty = \Phi$. Again we have frequencies of nullity.

WALLACE But that's wrong! In the limit, the frequencies, and hence the probabilities, should still be about $f = 1/2$ and $u = 9/10$.

JAMES My dear, dear friend. There you go again – mistaking a *limit* for a *value* at infinity. I just said that if I repeat the experiment lots of times then the ratios will come out about the same. In other words, if I carry out the experiment a number of times that has no real-numbered bound then, in the limit, the frequencies, and the probabilities, are as you expect. But if I carry out the experiment infinitely many times then the frequencies and probabilities are nullity. Infinity isn't unbounded, it is the biggest bound there is.

WALLACE Hmm. So you are saying that if I carry out an experiment any integer number of times, with $r = 1, 2, 3, \ldots$ then I get the frequencies and probabilities I expect, but if I carry out the experiment zero or infinitely often then I get frequencies and probabilities of nullity. In other words, a practical experiment produces a probability in the range zero to one, but an impractical experiment produces a probability of nullity. So, Einstein, what happens if I toss the coin nullity times?

JAMES Easy. When $r = \Phi$ it doesn't matter what the values of h and t are. The result is still a frequency, and a probability, of nullity. As you said, practical experiments produces a probability in the range zero to one, but impractical experiments produces a probability of nullity.

WALLACE What use is that?

JAMES Well, it means that there is always a frequency for something happening. That helps with software. If I haven't conducted a coin tossing experiment yet then a database of results might record $h = t = r = 0$ or $h = t = r = \Phi$, whichever I prefer. Now, if a standard computer attempts to calculate the frequency of a head or a tail it will crash. But a transreal computer just goes right ahead and calculates the frequencies as nullity. And I know what this means. It means an impractical experiment was run.

WALLACE Forget coins. Let's try something else. Before I hit my first golf ball, the chance of it dropping into the hole in one stroke is nullity, but after it has landed and come to rest, the frequency of a hole-in-one is somewhere in the range zero to one, no matter how often I go on to actually hit the ball?

JAMES Correct.

WALLACE And if God hits the golf ball infinitely often, the probability of a hole-in-one is nullity, even if He gets a hole in one on every stroke?

JAMES Correct. His probability is one in the limit at infinity and nullity at the value at infinity. There is a discontinuity in the probability distribution at infinity.

WALLACE Aha! You're wrong. You're wrong. I knew you were wrong! The exponential function is used all the time to describe probability distributions and it goes all the way to infinity!

JAMES Sure, but $e^{-\infty} = 0$, $e^{\infty} = \infty$, and $e^{\Phi} = \Phi$. I don't think you will ever find a problem with using the exponential as a probability distribution. Though I would have to ask my golf Partner to be sure.

WALLACE Ho, ho. That's a good one. Your golf partner. Next, you will be telling me that in a multiverse, the probability of a non-existent universe existing is nullity.

JAMES If you say so.

Contributors

James A.D.W. Anderson Author@bookofparagon.btinternet.co.uk