How to Divide by Zero In One Easy Lesson (And Some Hard Ones)

Dr. James A.D.W. Anderson
Computer Science
The University of Reading
England

Introduction

- A brief history of nothing
- Every computer in the world has a bug
- Standard forms – a hard lesson made easy
- Pandemonium and connecting chips together
- How to divide by zero in one easy lesson
- Pipelining
- Addition – a hard lesson
- Some hard questions

Introduction

- Why is infinity big?
- What is beyond the edge of the universe?
- What does division by zero mean for me?

A Brief History of Nothing

- Around 100 B.C. negative numbers are written about in China
- Brahmagupta (598 – 670 A.D.) writes about zero in India and gets it right. Also writes about dividing zero by zero, but gets it wrong
- James Anderson (1958 – A.D.) writes about the number nullity, which is zero divided by zero, in 1997 in Reading, England, and is proved correct in 2006 in Essex, England

A Bug in Every Computer

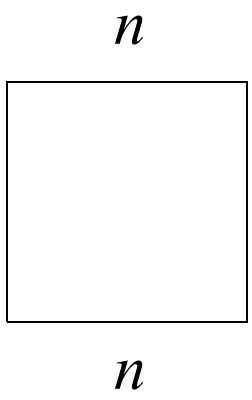
- Turn on your calculators or the calculator function in your mobile 'phone, watch, or pen
- Stand up
- Aisle 1 – Choose a positive number and divide it by zero.
- Aisle 2 – Divide zero by zero
- Aisle 3 – Choose a negative number and divide it by zero
- If your calculator shows an error then sit down

Standard Forms

- Any positive number divided by zero is *positive infinity*. Its standard form is $\infty = \frac{1}{0}$
- Zero divided by zero is nullity. It standard form is $\Phi = \frac{0}{0}$
- Any negative number divided by zero is negative infinity. Its standard form is $-\infty = \frac{-1}{0}$
- Rational and irrational numbers have their usual standard form

Pandemonium

perimeter $= 4n$



area $= n^2$

- Computer chips have connections only at the edge so they can transmit information only at the perimeter of the chip
- Sun Microsystems are experimenting with putting aerials in the surface of computer chips so they can transmit information everywhere in the area of a chip

Pandemonium

- As soon as you know the answer, clap!
- When you are asked for the answer, give it

Pandemonium

$$-\infty = \frac{-1}{0}; \qquad \Phi = \frac{0}{0}; \qquad \infty = \frac{1}{0}$$

The arithmetic mean is the sum of all data divided by the number of data

- What is the arithmetic mean of no data?

Pandemonium

$$-\infty = \frac{-1}{0}; \qquad \Phi = \frac{0}{0}; \qquad \infty = \frac{1}{0}$$

The probability of a coin showing a head is the number of times it shows a head, divided by the number of times it is tossed

- A coin lies heads up, but has never been tossed. What is the probability that it shows a head?

Pandemonium

$$-\infty = \frac{-1}{0}; \qquad \Phi = \frac{0}{0}; \qquad \infty = \frac{1}{0}$$

Ordinarily, the probability of a coin showing a tail is one minus the probability of it showing a head

- A coin lies heads up, but has never been tossed. What do you think the probability is that it shows a tail?

Dividing by zero

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

where $\frac{a}{b}, \frac{c}{d}, \frac{d}{c}$ are all in standard form

- Division by zero occurs when any of b, c, d is zero

Pipelining

Input devices:

- If you have some work and your *communicator* has no work then give your *communicator* one piece of work

Output devices:

- Your *communicator* believes you are a communicator
- If your *communicator* gives you one piece of work then hold it until you can pass on all the work you have

Pipelining

Communicators:

- Step 1 – If your *processor* has finished its work take its work and hold it until you can pass it on to the next *communicator*
- Step 2 – If you have some work and your *processor* has no work then give your work to your *processor*
- Step 3 – If you have some work and the next *communicator* has no work then pass your work onto the next *communicator*

Pipelining

Processors:

- When you are given some work to do, do it, then hold it up for your *communicator*

Pipelining

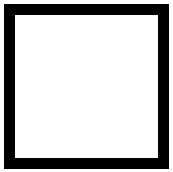
- Everyone in the first row and all subsequent odd rows, you are processors, sit still until you are given some work to do, then do it
- Everyone in the second row and all subsequent even rows, you are communicators. Your processor is immediately in front of you. Stand up
- I will pick input devices and output devices
- I will tell the input devices when to start

Pipelining

Reminder for Communicators:

- Step 1 – If your *processor* has finished its work take its work and hold it until you can pass it on to the next *communicator*
- Step 2 – If you have some work and your *processor* has no work then give your work to your *processor*
- Step 3 – If you have some work and the next *communicator* has no work then pass your work onto the next *communicator*

Work

- Add  to the running total
- The running total is 0,

Addition

All numbers are in standard form before they are added

- $\frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{\pm 1 \pm 1}{0}$ in particular

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ in general

Addition and Subtraction

Try working these out

- $0 + \infty$
- $3 + \infty$
- $\infty - 3$
- $\infty - 0$
- $\infty - \infty$

Multiplication and Division

Try working these out

- $0 \times \infty$
- $3 \times \infty$
- $\infty \div 3$
- $\infty \div 0$
- $\infty \div \infty$

Hard Problems

Can you work these out:

- $\sqrt{\Phi}$
- $\sqrt{\infty}$
- 0^0
- 1^x where x is a real number or $\pm\infty$ or Φ
- Why is it the case that $\cos^2 x + \sin^2 x = 1^x$

Infinity is Big

- We define that $a > b$ when $a - b > 0$
- We define that $\infty > 0$ and $-\infty < 0$
- We define that any number a obeys exactly one of $a < 0, a = 0, a > 0, a = \Phi$
- We note that the standard form of real numbers, $\frac{n}{d}$, has $d > 0$
- We note that the standard form of transreal numbers, $\frac{n}{d}$, has $d \geq 0$

Infinity is Big

Prove that infinity is big by proving that:

- Positive infinity is bigger than any real number
- Positive infinity is bigger than negative infinity
- Positive infinity is not bigger than itself
- Nullity is not bigger than positive infinity

Infinity is Big

- Positive infinity is bigger than any real number, with denominator $d > 0$, because:

$$\infty - \frac{n}{d} = \frac{1}{0} - \frac{n}{d} = \frac{1 \times d - n \times 0}{0 \times d} = \frac{d}{0} = \frac{1}{0} = \infty > 0$$

Infinity is Big

- Positive infinity is bigger than negative infinity because:

$$\infty - (-\infty) = \frac{1}{0} - \frac{-1}{0} = \frac{1}{0} + \frac{1}{0} = \frac{1+1}{0} = \frac{2}{0} = \frac{1}{0} = \infty > 0$$

Infinity is Big

- Positive infinity is not bigger than itself because:

$$\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{1-1}{0} = \frac{0}{0} = \Phi$$

- And nullity is not greater than zero

Infinity is Big

- Nullity is not bigger than positive infinity because:

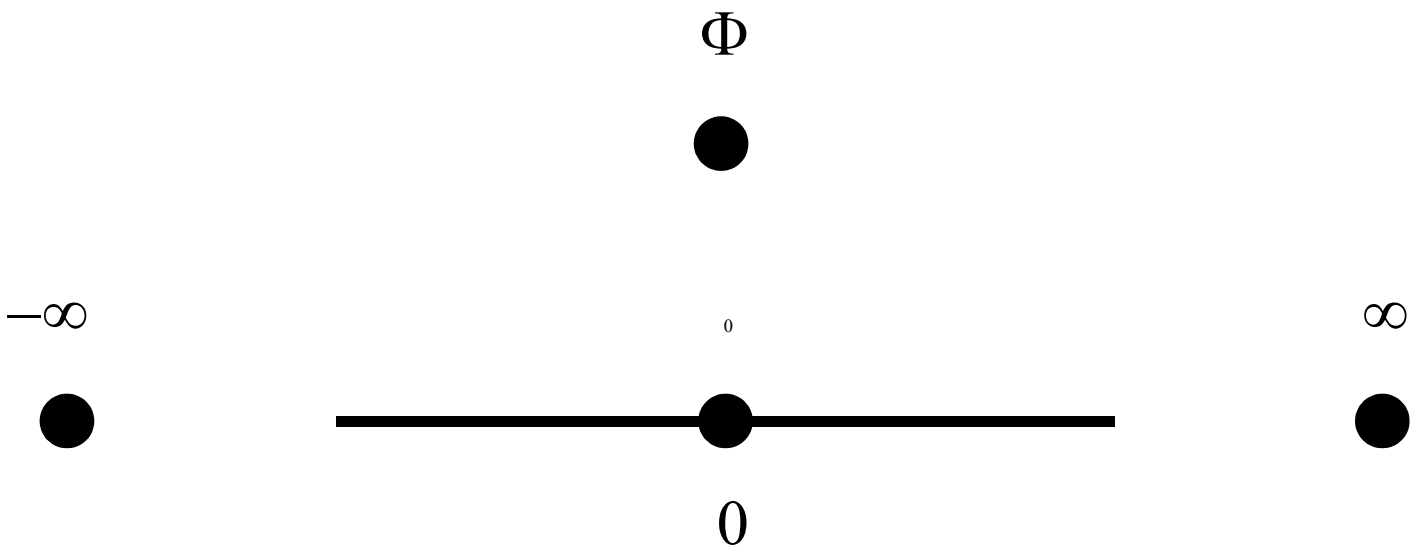
$$\Phi - \infty = \frac{0}{0} - \frac{1}{0} = \frac{0 \times 0 - 1 \times 0}{0 \times 0} = \frac{0}{0} = \Phi$$

- And nullity is not greater than zero

Infinity is Big

- It is impossible to prove that infinity is big using ordinary arithmetic
- It is impossible to prove that infinity is big using ordinary algebra
- It is impossible to prove that infinity is big using ordinary calculus
- But you can prove that infinity is big because you can divide by zero

Beyond the Edge



What does this mean for teachers?

- When pupils ask how to divide by zero, you can show them
- You can show your pupils some fun mathematics
- You might consider how division by zero changes your teaching
- If computing or physics makes heavy use of division by zero, you will be ready to teach division by zero in primary and secondary schools

What does this mean for pupils?

- All of the positive results of mathematics you have been taught remain true
- Some of the negative results of mathematics you have been taught become false
- Many of the results of mathematics you have been taught become unnecessarily limited
- You might have to learn how to divide by zero in order to use computers that are more advanced than today's computers
- You might have to learn how to divide by zero in the physical sciences

What does this mean for pupils?

- WARNING - there is a lot more to know about division by zero than I have told you today. If you try to use it you might go wrong …
- … or you might become a famous mathematician by being one of the first people to research the number nullity