How to Divide by Zero (and survive the experience)

Gillotts Annual Specialist School Lecture 2009

Dr James A.D.W. Anderson

Introduction

During this talk I will:

- Tell you about my life and how I became a scientist
- Tell you how I discovered how to divide by zero, using only pre-existing algorithms of arithmetic
- Tell you how I dealt with the backlash against my ideas, how I learned from my critics, and how most of them failed to learn from me
- Tell you how you can make discoveries in mathematics and science

But, first, an adventure!

Can You Divide by Zero?

$$1 \div 1 = 1 \qquad -1 \div 1 = -1$$

$$1 \div 0.1 = 10 \qquad -1 \div 0.1 = -10$$

$$1 \div 0.01 = 100 \qquad -1 \div 0.01 = -100$$

$$1 \div 0.001 = 1000 \qquad -1 \div 0.001 = -1000$$

$$1 \div 0 = ? \qquad -1 \div 0 = ?$$

$$0 \div 0 = ?$$

Can Calculators Divide by Zero?

- If you have an electronic calculator with you then turn it on and stand up
- Pick a number and divide it by zero on your calculator
- If your calculator shows an error or has crashed then sit down
- If your calculator is still working then multiply the current answer by zero
- If your calculator shows an error or has crashed then sit down
- Is there anyone left standing?

Can Computers Divide by Zero?

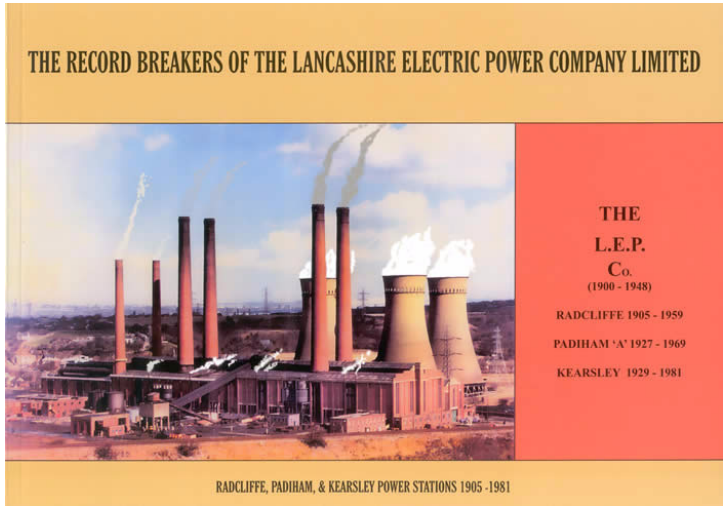
- The bridge of the missile cruiser, USS Yorktown, had networked computer control of navigation, engine monitoring, fuel control, machinery control, and damage control

Can Computers Divide by Zero?

- On September 21st, 1997, a sailor on the USS Yorktown entered a zero into a database field, causing a division by zero error which cascaded through the ship's network, crashing every computer on the network, and leaving the ship dead in the water for 2 hours 45 minutes
- The world would be a safer place if computers, calculators and people could divide numbers by zero, getting a number as an answer
- Coincidentally, I worked out how to do this in 1997

Autobiography

- My parents are now dead. When I was a young child my father worked shifts in a power station. My parents often argued when he came home from work. I put the television on so that I could not hear them. I watched programs on science, engineering, and mathematics. My father went to night school and became a statistician working for an electricity board



My father worked at Padiham A

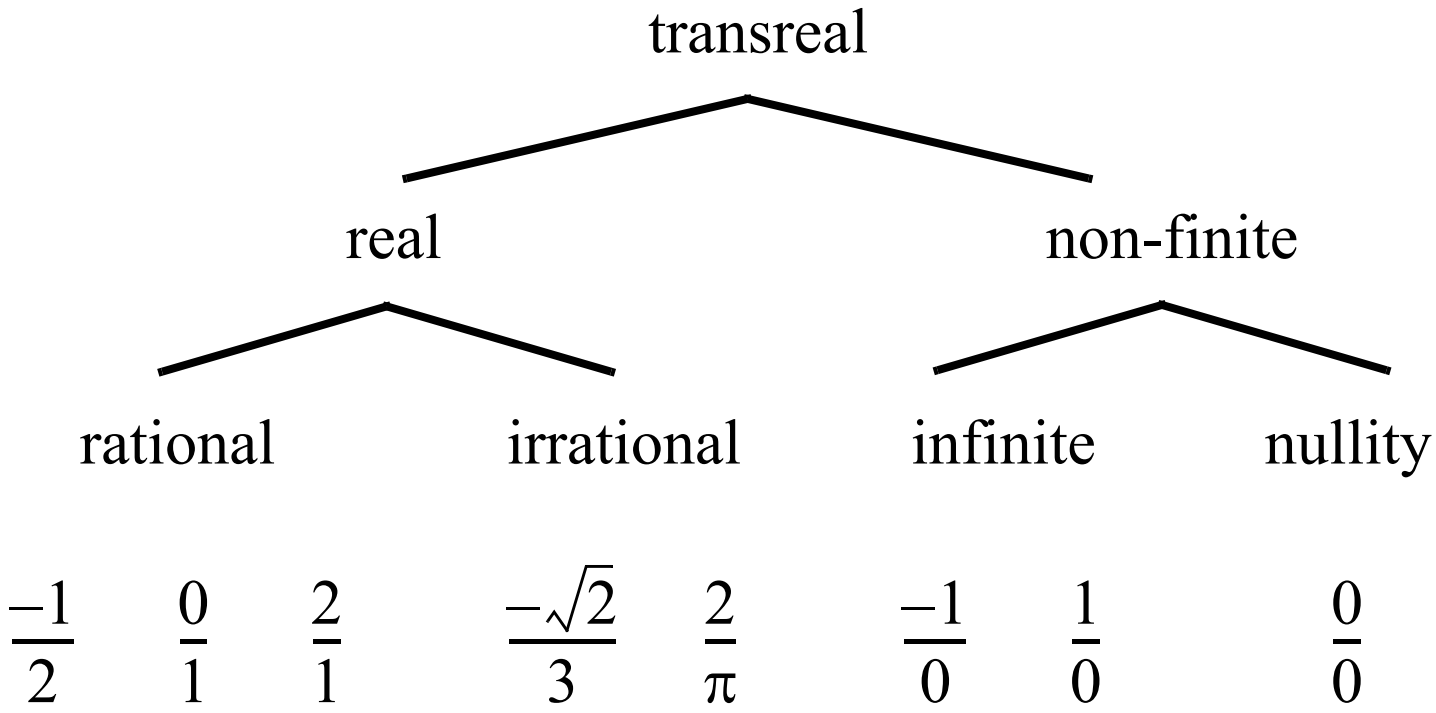
Autobiography

- I used to walk several miles to my primary school, in the next village, and think about the things I had learned from television, before doing my school lessons

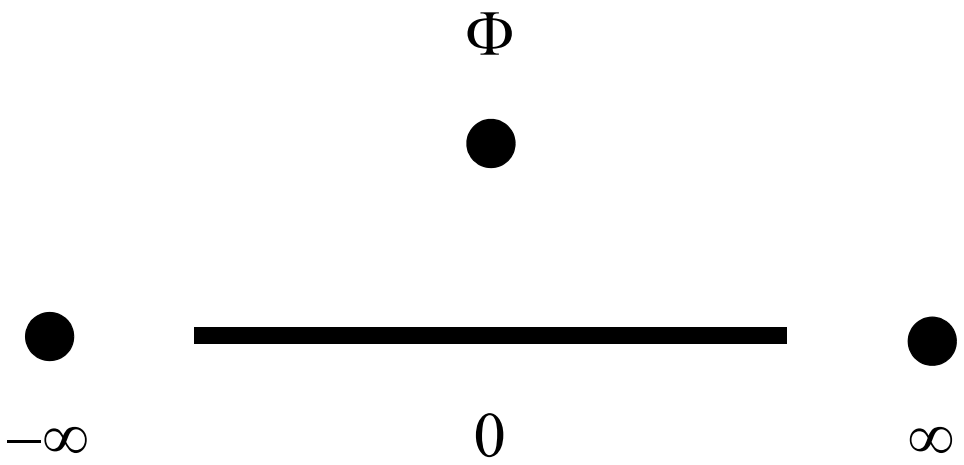
- I will now give you the experience of advanced learning on TV and elementary learning at school

TV – Transreal Numbers

- The transreal numbers are all of the real numbers, as used in everyday life, together with three non-finite numbers



TV – Transreal Numbers



- Positive infinity, ∞, is the biggest transreal number
- Negative infinity, −∞, is the smallest transreal number
- Nullity, Φ, is the only transreal number that is not negative, not zero, and not positive

TV – Transreal Numbers

- Positive Infinity, ∞, is any positive number divided by zero

  Its standard form is $\infty = \frac{1}{0}$

- Negative infinity, $-\infty$, is any negative number divided by zero

  Its standard form is $-\infty = \frac{-1}{0}$

TV – Transreal Numbers

- Nullity, Φ, is zero divided by zero

  Its standard form is $\Phi = \frac{0}{0}$

- The fraction zero, 0, is the integer zero, 0, divided by any non-zero number

  Its standard form is $0 = \frac{0}{1}$

TV – Transreal Fractions

A *transreal number* is a *transreal fraction* of the form $\frac{n}{d}$, where:

- n is the *numerator* of the fraction
- d is the *denominator* of the fraction
- n, d are *real numbers*
- $d \geq 0$
- Examples: $\frac{-1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{-\pi}{2}, \frac{-1}{2\pi}, \frac{1}{2\pi}, \frac{-1}{0}, \frac{1}{0}, \frac{0}{0}$

TV – Transreal Fractions

- An *improper transreal fraction,* $\frac{n}{-d}$, may have a negative denominator, $-d < 0$
- An improper transreal fraction is converted to a *proper transreal fraction* by multiplying both the numerator and the denominator by -1
- Example: $\frac{2}{-3} = \frac{-1 \times 2}{-1 \times (-3)} = \frac{-2}{3}$
- Example: $\frac{0}{-1} = \frac{-1 \times 0}{-1 \times (-1)} = \frac{0}{1}$

Autobiography

- I am dyslexic. I could not read at all until the age of 9. I could not read well until the age of 30. I am now 50 and would like to learn more about writing
- When I was at school, I would take what my teachers said very literally. In later life this made me a very critical reader of scientific literature. Evidently, more critical than most of my critics
- At school, I developed a very good memory for the spoken word and pictures, but I still find it hard to remember things that are written down. This forces me to re-read things. This is very good, because it means I regularly check definitions and the primary scientific literature. Most of my critics, evidently, do not check

School – Multiplication

Two fractions are multiplied like this:

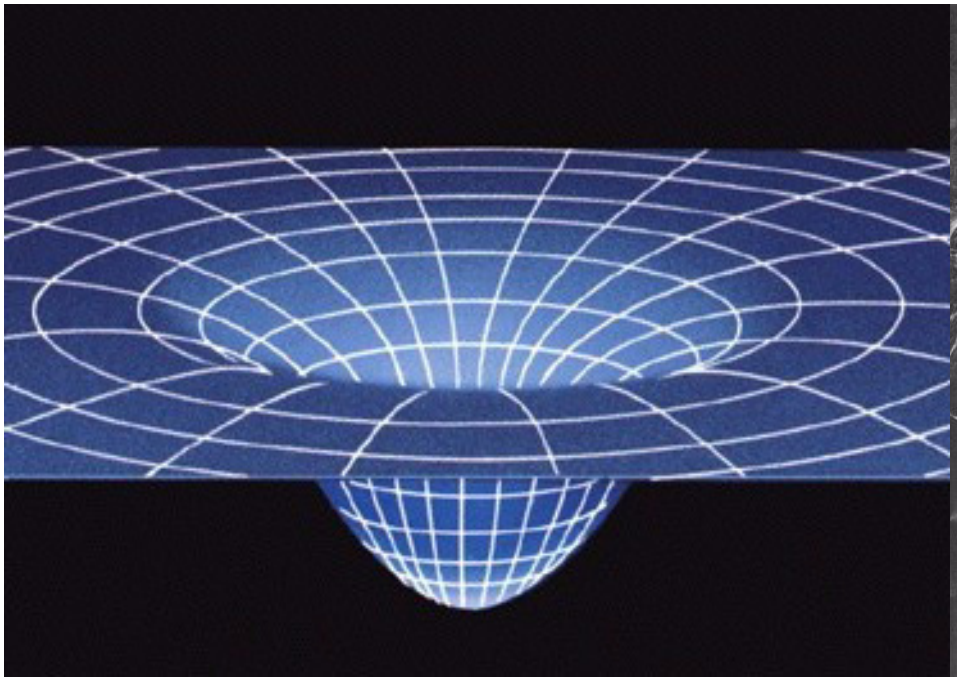
- $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

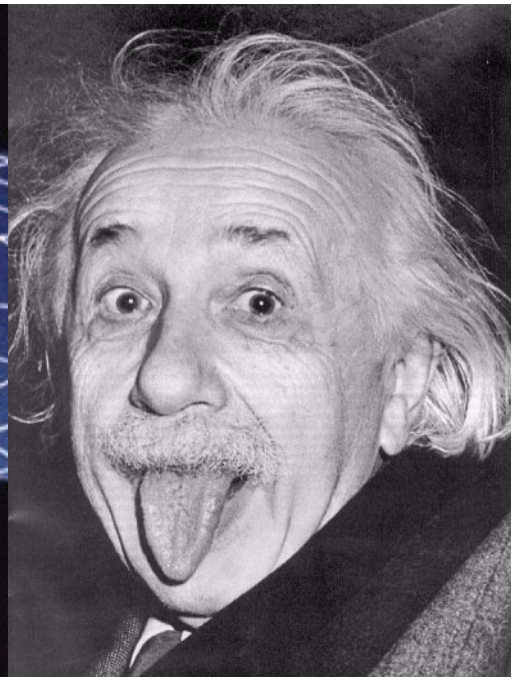
- What have you learned from this? I learned how to divide by zero, because my teacher did not say that $b, d \neq 0$

- But I did not realise I had learnt how to divide by zero until much later. I keep a log book and re-read it from time to time

Albert Einstein

- Albert Einstein discovered the curvature of spacetime two years before he realised he had. Fortunately, Einstein was dyslexic and kept a note book. He looked up the answer when he needed it





Transreal Multiplication

Two *proper transreal fractions* are multiplied like this:

- $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

- Example: $3 \times \infty = \frac{3}{1} \times \frac{1}{0} = \frac{3 \times 1}{1 \times 0} = \frac{3}{0} = \infty$

- Example: $0 \times \infty = \frac{0}{1} \times \frac{1}{0} = \frac{0 \times 1}{1 \times 0} = \frac{0}{0} = \Phi$

- Example: $-3 \times \infty = \frac{-3}{1} \times \frac{1}{0} = \frac{-3 \times 1}{1 \times 0} = \frac{-3}{0} = -\infty$

Transreal Division

Two *proper transreal fractions* are divided like this:

- $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

- Example: $$\infty \div 3 = \frac{1}{0} \div \frac{3}{1} = \frac{1}{0} \times \frac{1}{3} = \frac{1 \times 1}{0 \times 3} = \frac{1}{0} = \infty$$

- Example:

$$\infty \div (-3) = \frac{1}{0} \div \frac{-3}{1} = \frac{1}{0} \times \frac{1}{-3} = \frac{1}{0} \times \frac{-1 \times 1}{-1 \times (-3)}$$

$$= \frac{1}{0} \times \frac{-1}{3} = \frac{1 \times (-1)}{0 \times 3} = \frac{-1}{0} = -\infty$$

Transreal Addition

Two *proper transreal fractions* are added like this:

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, except that:

- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Transreal Addition

- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Examples:

- $\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{1 + 1}{0} = \frac{2}{0} = \infty$
- $(-\infty) + (-\infty) = \frac{-1}{0} + \frac{-1}{0} = \frac{(-1) + (-1)}{0} = \frac{-2}{0} = -\infty$
- $\infty + (-\infty) = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{0}{0} = \Phi$

Transreal Addition

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Examples:

- $\frac{2}{3} + \infty = \frac{2}{3} + \frac{1}{0} = \frac{2 \times 0 + 3 \times 1}{3 \times 0} = \frac{3}{0} = \infty$
- $\frac{2}{3} + \Phi = \frac{2}{3} + \frac{0}{0} = \frac{2 \times 0 + 3 \times 0}{3 \times 0} = \frac{0}{0} = \Phi$
- $\frac{2}{3} + \frac{4}{5} = \frac{2 \times 5 + 3 \times 4}{3 \times 5} = \frac{22}{15}$

Transreal Subtraction

Two *proper transreal fractions* are subtracted like this:

- $$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

Examples:

- $$\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{1 - 1}{0} = \frac{0}{0} = \Phi$$
- $$\frac{1}{2} - \frac{3}{5} = \frac{1}{2} + \frac{-3}{5} = \frac{(1 \times 5) + (2 \times (-3))}{2 \times 5} = \frac{5 + (-6)}{10}$$
  $$= \frac{-1}{10}$$

Challenge

The number zero was discovered over 1 300 years ago

A question for the Philosophy of Science:

- Why did it take 65 generations of mathematicians, until me, to realise that the operations of arithmetic allow division by zero?

A question for the History of Science:

- How close did previous generations come to realising that the operations of arithmetic allow division by zero?

Now the *really* tough question:

Challenge

A question that is very hard to ask, and even harder to answer:

- Why didn't you work it out?

Autobiography

- My grandfather was a minister. I went to a school administered by his church. My grandfather gave me some of his old books on mathematics, chemistry, physics, psychology, and religion
- The mathematics I learnt in the books was different from the mathematics I leant in school
- The physics of motion was the same, but there was no mention of nuclear physics in the books
- Some of the chemistry in the books was wrong, it made unwarranted assumptions
- Many of the old books made unwarranted assumptions

Autobiography

- I became critical of received knowledge and chose to read about the history of science. I read about the great, and the infamous, scientists. Occasionally, I read autobiographies of mundane scientists. I learnt everyone's methods, how they did science, and how they dealt with a world that sometimes treated them very harshly

Autobiography

- One day I made a bad mistake in a chemistry class. I watched smoke particles moving randomly under a microscope. The teacher asked what we saw. The first student said that all the particles moved in one direction. In turn, every student agreed. The teacher asked me, last, what I had seen. I said that the particles moved in one direction, even though I knew they had not. The teacher said they moved randomly and made us all look at the smoke again

- I learnt more from that mistake than from my many, very accurate, experiments in chemistry, but I learnt a lot of chemistry too

- In later life I became an experimental psychologist

Research

- But, fortunately, I have a terrible bedside manner so I gave up psychology and became a computer scientist
- In 1997 I discovered the transreal numbers as a solution to a practical problem in trigonometry. I explored many ways of using these numbers in computers, which I published and lectured on
- In January 2006, I gave a seminar on transreal numbers at Essex University. My audience asked if I had a computer proof of the consistency of division by zero. This is the most detailed kind of proof it is possible to have. I said that I did not. One of the audience offered to develop such a proof, if I would axiomatise transreal arithmetic

Backlash

- In June 2006 papers were pre-published giving the machine proof of division by zero and its application to transreal analysis

- Then the world went mad. 100 000 people flamed me on the web, 1 000 people emailed or telephoned me, most to say that I was wrong, mad, dangerous, or evil

Giordano Bruno

Giordano Bruno taught that Nicolaus Copernicus was right to believe that the Earth moves round the sun. Bruno was burnt at the stake, for various heresies, in 1600

Being flamed on the web is a big improvement on being burnt at the stake!

Backlash

- Three BBC journalists checked the division by zero story with me, one journalist each from the USA, Scandinavia, India, and China also checked the story with me. No one else checked with me, though 40 000 people downloaded the axioms paper in a week

- Only one person referred to the computer proof of consistency, only to say that there was no detectable error in it, apart from a typographical error in theorem 53 – after the proof

Augustin Cauchy

Surveys have found that 80% of mathematics papers contain errors. Augustin Cauchy's work on complex analysis (calculus) repeated errors for several years

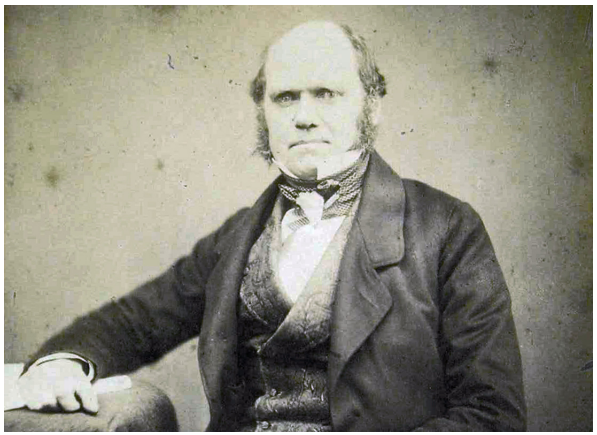
Making errors is part of the human condition, correcting them is part of science

Backlash

- It is very, very, very important to read primary sources and to check definitions
- If my critics had read my papers, or checked the definitions they used in their criticisms of me, then I would have had only a few critics

Mendel and Darwin

- Gregor Mendel (left) discovered discrete genetic inheritance in the mid 19th Century at the same time as Charles Darwin (right) was criticised for not having such a theory



- Contemporary scientists cited Mendel's work, but chose not to read it in the Vatican library. It was rediscovered in the 20th century by a scientist who read the primary source – Mendel's papers

Backlash

- In a space of two to three weeks, I answered all of the people who contacted me directly, to point out the errors in their criticisms. One, a professional algebraist, acknowledged that I was correct. One amateur and one professional mathematician contacted me to claim that they had invented the transreal numbers before me. The amateur did not acknowledge my counter proof, the professional did, and withdrew the claim. No one else acknowledged receipt of my replies, except to defame or threaten me

- Wikipedia ran many stories on my work, without reporting the primary source of the mathematical proof of consistency, and denied me the opportunity to correct their stories – except to remove a libel

Return of the Scientist

- Darwin used to keep a record of his critics' views. He thought it was easy to remember one's own theories, but helpful to reconsider the theories of one's opponents

- Four of the criticisms of my work were quite deep. I dealt with two in a paper on the topology of transreal numbers. I dealt with one in the development of teaching materials for secondary schools, to be used in academic research by professional educationalists, and I have dealt with one in a paper on the polar-transcomplex numbers which has yet to be published. Thus, in 2009, I have answers to all of the criticisms made in 2006, though not all of my answers are publicly available yet

New, New Mathematics

In Britain, it took about 100 years to replace Roman numerals with Arabic ones. This transformation was carried out mainly by tradespeople, not by the academic authorities of the time. How long will it take to accept transreal numbers?

People work for about 40 years – about two generations

- I invented the transreal numbers in 1997. I hope to establish a bridgehead in science, commerce, education, and public knowledge over a space of 20 years to 2017. I am due to retire in 2024

New, New Mathematics

- I hope that a majority of university lecturers, in relevant subjects, and relevant specialists in commerce, engineering, and education will be using transreal numbers in 2037, one generation after the bridgehead

- I hope that the majority of university lecturers, in relevant subjects, will introduce transreal arithmetic to secondary schools in 2037. It will then take two generations, to 2077, for all secondary school teachers to be using transreal arithmetic. At which point all of the population of working age will be using transreal arithmetic

- This is 80 years, a saving of 20 years on the introduction of Arabic numerals

What Can You Do?

- Transreal arithmetic contains real arithmetic and is consistent with it. You will not do yourself any harm by learning transreal arithmetic
- Transreal arithmetic is very new, it is within your ability to make mathematical discoveries that no one else has made
- It is difficult to make useful discoveries, but here are a few tips …

How to Discover Useful Things

- Read primary scientific sources, keep a journal, and re-read your journal from time to time
- Learn something of the history and philosophy of science and read biographies of scientists
- Learn as much science as you can
- Work on problems continually, until you can keep the whole problem in your mind and can work on it during quiet moments of the day
- Think about scientific problems as you go to sleep, during your dreams, and as you wake up

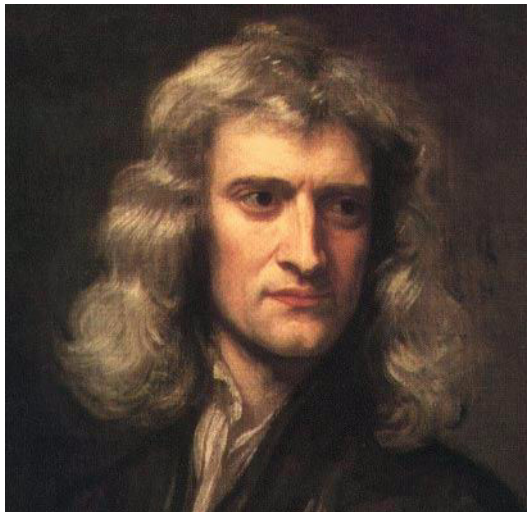
How to Discover Useful Things

- As Charles Darwin said, “I am turned into a sort of machine for observing facts & grinding out conclusions”
- Discuss your inventions with many people, record their criticisms in your journal, and work out answers to their criticisms. Record your answers in your journal and share them with your critics
- Work out how your discoveries might be used to make the world a better place, and record your proposals in your journal

How to Discover Useful Things

- Record anything that takes your fancy. I have proposals for two time machines, a sentient robot, a very fast computer, a method for dividing by zero, and a very mathematical use for a walnut cake

- As Isaac Newton said, “No great discovery was ever made without a bold guess”

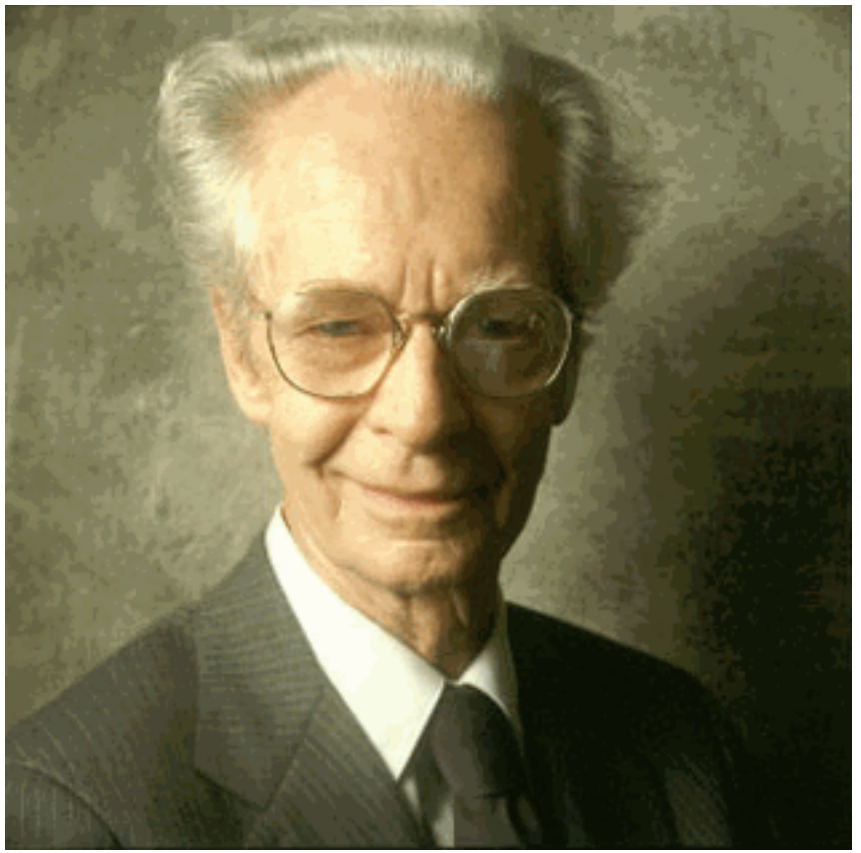


How to Discover Useful Things

- Try to be polite and gentle with your critics. As Isaac Newton also said, “Tact is the art of making a point without making an enemy”
- Use pictures to explain things, it helps the dyslexic and the innumerate
- Follow Albert Einstein’s advice to find a scientific goal that fascinates you, and pursue it without regard to your scientific standing and career

How to Discover Useful Things

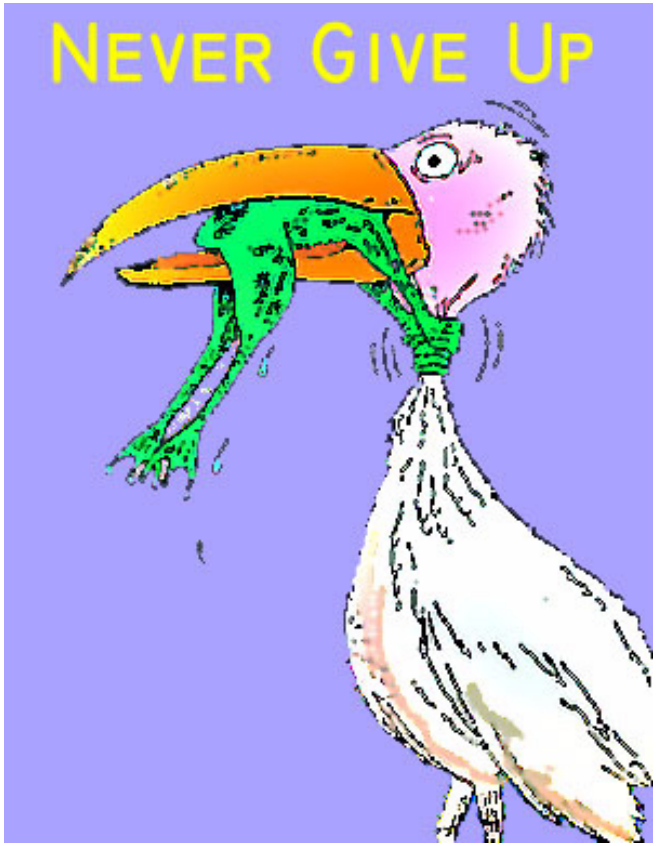
- But also follow Burrhus Skinner's advice that when you see the main chance, give everything up and pursue it



How to Discover Useful Things

Finally, as Winston Churchill said, at a school prize giving day at Harrow, “Never give in. Never give in. Never, never, never, never – in nothing, great or small, large or petty – never give in, except to convictions of honour and good sense. Never yield to force. Never yield to the apparently overwhelming might of the enemy”

Never Give Up!



For more information about transreal mathematics see:
http://www.bookofparagon.com