Transreal Arithmetic

IBM Blue Fusion
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Opportunity

- Our lives depend on computers working correctly
- Computers must always know how to carry out a calculation otherwise they fail
- But the arithmetic you have learnt in school cannot divide by zero, it forces computers to fail

Activity – Calculators

- If you have an electronic calculator with you then turn it on and stand up
- Pick a number and divide it by zero on your calculator
- If your calculator shows an error or has crashed then sit down
- If your calculator is still working then multiply the current answer by zero
- If your calculator shows an error or has crashed then sit down
- Is there anyone left standing?

USS Yorktown

The bridge of the missile cruiser, USS Yorktown, had networked computer control of navigation, engine monitoring, fuel control, machinery control, and damage control

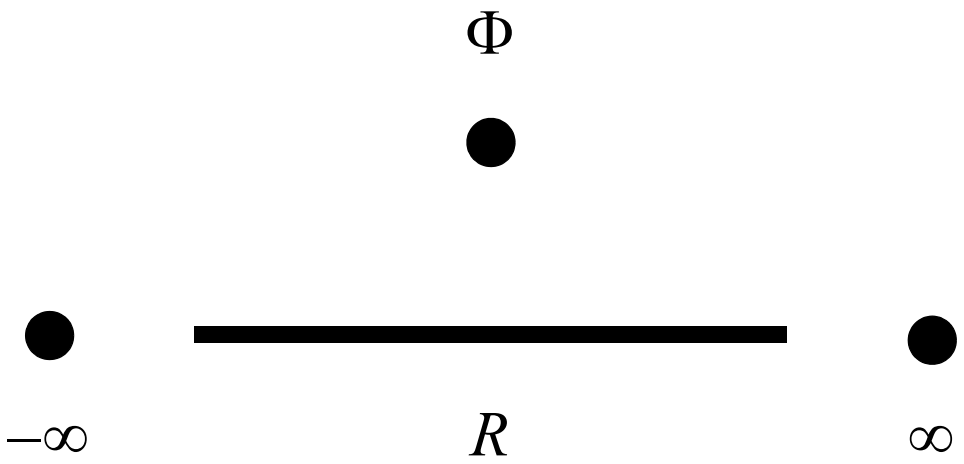
USS Yorktown

On September 21st, 1997, a crew member entered a zero into a database field, causing a division by zero error which cascaded through the ship's network, crashing every computer on the network, and leaving the ship dead in the water for 2 hours 45 minutes

Transreal Arithmetic

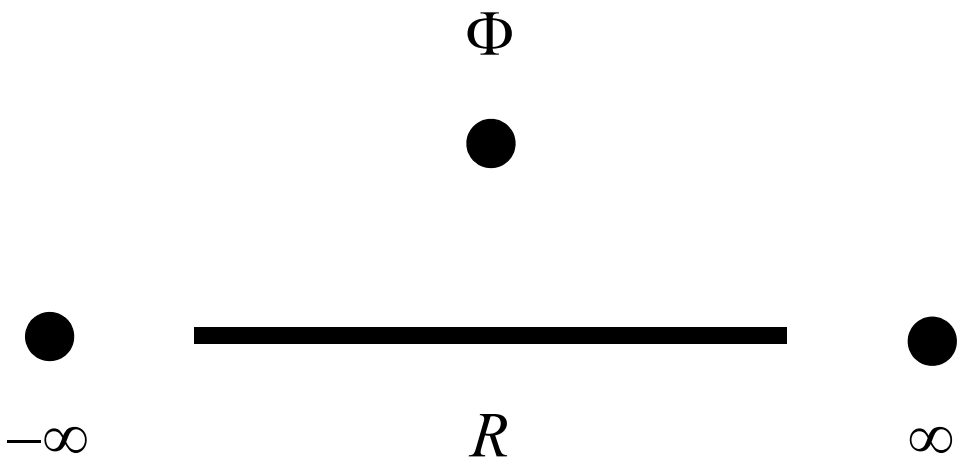
- The arithmetic you have learnt in school works on *real* numbers
- There are many other kinds of numbers
- You are now going to learn about *transreal* numbers that do everything that real numbers do and more
- Transreal numbers can divide by zero
- Computers programmed with transreal numbers never fail when performing an arithmetical operation – though they might lose accuracy

Transreal Number Line



- R is the *real-number line.* It holds all of the numbers you have learnt in school
- ∞ is *positive infinity* and $-\infty$ is *negative infinity*
- Φ is *nullity*

Transreal Number Line



- Positive infinity, ∞, is the biggest transreal number
- Negative infinity, $-\infty$, is the smallest transreal number
- Nullity Φ is the only transreal number that is not negative, not zero, and not positive

Transreal Numbers

- Positive Infinity, ∞, is any positive number divided by zero

  Its standard form is $\infty = \frac{1}{0}$

- Negative infinity, $-\infty$, is any negative number divided by zero

  Its standard form is $-\infty = \frac{-1}{0}$

Transreal Numbers

- Nullity, Φ, is zero divided by zero

  Its standard form is $\Phi = \frac{0}{0}$

- The fraction zero, 0, is the integer zero, 0, divided by any non-zero number

  Its standard form is $0 = \frac{0}{1}$

Transreal Fractions

A *transreal number* is a *transreal fraction* of the form $\frac{n}{d}$, where:

- n is the *numerator* of the fraction
- d is the *denominator* of the fraction
- n, d are *real numbers*
- $d \geq 0$
- Examples: $\frac{1}{2}, \frac{-1}{2}, \frac{\pi}{2}, \frac{-\pi}{2}\ \frac{1}{2\pi}, \frac{-1}{2\pi}, \frac{-1}{0}, \frac{0}{0}, \frac{1}{0}$

Transreal Fractions

- An *improper transreal fraction,* $\frac{n}{-d}$, may have a negative denominator, $-d < 0$
- An improper transreal fraction is converted to a *proper transreal fraction* by multiplying both the numerator and the denominator by -1
- Example: $\frac{2}{-3} = \frac{-1 \times 2}{-1 \times (-3)} = \frac{-2}{3}$
- Example: $\frac{0}{-1} = \frac{-1 \times 0}{-1 \times (-1)} = \frac{0}{1}$

Transreal Multiplication

Two proper transreal fractions are multiplied like this:

- $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

- Example: $3 \times \infty = \frac{3}{1} \times \frac{1}{0} = \frac{3 \times 1}{1 \times 0} = \frac{3}{0} = \infty$

- Example: $0 \times \infty = \frac{0}{1} \times \frac{1}{0} = \frac{0 \times 1}{1 \times 0} = \frac{0}{0} = \Phi$

- Example: $-3 \times \infty = \frac{-3}{1} \times \frac{1}{0} = \frac{-3 \times 1}{1 \times 0} = \frac{-3}{0} = -\infty$

Transreal Division

Two *proper transreal fractions* are divided like this:

- $$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

- Example: $$\infty \div 3 = \frac{1}{0} \div \frac{3}{1} = \frac{1}{0} \times \frac{1}{3} = \frac{1 \times 1}{0 \times 3} = \frac{1}{0} = \infty$$

- Example:

$$\infty \div (-3) = \frac{1}{0} \div \frac{-3}{1} = \frac{1}{0} \times \frac{1}{-3} = \frac{1}{0} \times \frac{-1 \times 1}{-1 \times (-3)}$$

$$= \frac{1}{0} \times \frac{-1}{3} = \frac{1 \times (-1)}{0 \times 3} = \frac{-1}{0} = -\infty$$

Transreal Addition

Two *proper transreal fractions* are added like this:

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$, except that:

- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Transreal Addition

- $(\pm\infty) + (\pm\infty) = \frac{\pm 1}{0} + \frac{\pm 1}{0} = \frac{(\pm 1) + (\pm 1)}{0}$

Examples:

- $\infty + \infty = \frac{1}{0} + \frac{1}{0} = \frac{1 + 1}{0} = \frac{2}{0} = \infty$
- $(-\infty) + (-\infty) = \frac{-1}{0} + \frac{-1}{0} = \frac{(-1) + (-1)}{0} = \frac{-2}{0} = -\infty$
- $\infty + (-\infty) = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{0}{0} = \Phi$

Transreal Addition

- $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

Examples:

- $\frac{2}{3} + \infty = \frac{2}{3} + \frac{1}{0} = \frac{2 \times 0 + 3 \times 1}{3 \times 0} = \frac{3}{0} = \infty$
- $\frac{2}{3} + \Phi = \frac{2}{3} + \frac{0}{0} = \frac{2 \times 0 + 3 \times 0}{3 \times 0} = \frac{0}{0} = \Phi$
- $\frac{2}{3} + \frac{4}{5} = \frac{2 \times 5 + 3 \times 4}{3 \times 5} = \frac{22}{15}$

Transreal Subtraction

Two *proper transreal fractions* are subtracted like this:

- $$\frac{a}{b} - \frac{c}{d} = \frac{a}{b} + \frac{-c}{d}$$

Examples:

- $$\infty - \infty = \frac{1}{0} - \frac{1}{0} = \frac{1}{0} + \frac{-1}{0} = \frac{1 + (-1)}{0} = \frac{1 - 1}{0} = \frac{0}{0} = \Phi$$
- $$\frac{1}{2} - \frac{3}{5} = \frac{1}{2} + \frac{-3}{5} = \frac{(1 \times 5) + (2 \times (-3))}{2 \times 5} = \frac{5 + (-6)}{10}$$
$$= \frac{-1}{10}$$

Conclusion

- Everything you learnt in mathematics lessons at school is true of real numbers
- You have now learnt something about transreal numbers. But there is more to know than you have learnt today
- Transreal arithmetic contains real arithmetic and is consistent with it so you will not come to any harm if you use the arithmetic you have learnt today

Activity – Parallel Computing

- When you receive a piece of paper with a program on it, do what it says and pass it on to your neighbour

Activity – Parallel Computing

- You have just been a parallel computer with 90 of you doing real arithmetic
- Computer manufacturers can design computer chips with 80 cores doing real arithmetic
- I can design a computer chip with 4 000 cores doing transreal arithmetic
- Watch the movie of the real supercomputer and of the transreal computer to see how fast it works