Construction of the Transcomplex Numbers

Prof. Tiago S dos Reis
Dr James Anderson FBCS CITP CSci
Agenda

• Advantages of transcomplex numbers
• Containment relationships
• Construction of the transcomplex numbers
• Value to science and society
Advantages

• Transcomplex arithmetic builds on the foundation of transreal arithmetic

• Consistency of transcomplex and transreal numbers proved by construction from the complex numbers

• Transcomplexes allow the solution of mathematical and physical problems at singularities

• Transcomplexes make mathematical software more reliable
Overview
Transreal Number Line

\[ -\infty \Phi \infty \]
Transcomplex Plane
Revolution of the transreal number line
Construction

• Aims to give sense to dividing complex numbers by zero
Construction

\[ T := \{(x, y); \ x \in \mathbb{C}, \ y \in \{0, 1\}\} \]
Construction

\[ T := \{(x, y); \ x \in \mathbb{C}, \ y \in \{0,1\}\} \]

\[ (x, y) \sim (w, z) \]

\[ \exists \alpha \in \mathbb{R}^+; \ x = \alpha w \ \text{and} \ y = \alpha z \]
Construction

Reflexive

\((x, y) \sim (x, y)\)
Construction

Reflexive

\[(x, y) \sim (x, y)\]

Symmetric

\[(x, y) \sim (w, z) \Rightarrow (w, y) \sim (x, y)\]
Construction

Reflexive

\[(x, y) \sim (x, y)\]

Symmetric

\[(x, y) \sim (w, z) \implies (w, y) \sim (x, y)\]

Transitive

\[(x, y) \sim (w, z) \text{ and } (w, z) \sim (u, v) \implies (x, y) \sim (u, v)\]
Construction

\[ C^T := T \sim \]
Construction

Addition

If \([x, y], [w, z] \in \{[u, 0]; u \in \mathbb{C} \setminus \{0\}\}\):

\[
[x, y] + [w, z] := \left[ \frac{x}{|x|} + \frac{w}{|w|}, 0 \right]
\]

otherwise,

\[
[x, y] + [w, z] := [xz + wy, yz]
\]
Construction

Multiplication

\([x, y] \times [w, z] := [xw, yz]\)
Construction

Multiplication

\([x, y] \times [w, z] := [xw, yz]\)

Opposite

\([-[x, y] := [-x, y]\)
Construction

Multiplication

\([x, y] \times [w, z] := [xw, yz]\)

Opposite

\(-[x, y] := [-x, y]\)

Reciprocal

\([x, y]^{-1} := \begin{cases} \left[ \frac{y}{x}, 1 \right], & x \neq 0 \\ [y, x], & x = 0 \end{cases}\)
Construction

Subtraction

\[[x, y] - [w, z] := [x, y] + (-[w, z])\]
Construction

Subtraction

\[ [x, y] - [w, z] := [x, y] + (\neg [w, z]) \]

Division

\[ [x, y] \div [w, z] := [x, y] \times [w, z]^{-1} \]
Construction

\[ C^T = \{ [x,1]; x \in \mathbb{C} \} \cup \{ [w,0]; w \in \mathbb{C}, |w|=1 \} \cup \{ [0,0] \} \]
Construction

$\{[x,1]; x \in \mathbb{C}\}$ is a copy of $\mathbb{C}$
Construction

\[ \{ [x,1]; x \in \mathbb{C} \} \text{ is a copy of } \mathbb{C} \]

\[ \mathbb{C} = \{ [x,1]; x \in \mathbb{C} \} \]
Construction

\[
\{ [x,1]; x \in \mathbb{C} \} \text{ is a copy of } \mathbb{C}
\]

\[
\mathbb{C} = \{ [x,1]; x \in \mathbb{C} \}
\]

\[ x = [x,1] \]
Construction

\[ \{[x,1]; x \in \mathbb{C}\} \text{ is a copy of } \mathbb{C} \]

\[ \mathbb{C} = \{[x,1]; x \in \mathbb{C}\} \]

\[ x = [x,1] \]

\[ [x,y] = \frac{x}{y} \]
Construction

\((r, \theta)\)
Construction

\[(r, \theta) = \begin{cases} [x, 1] \\ [u, 0] \\ [0, 0] \end{cases}\]
Construction

\[(r, \theta) = \begin{cases} 
[x, 1], & r = |x| \text{ and } \theta = \text{Arg}(x) \\
[u, 0] \\
[0, 0] 
\end{cases}\]
Construction

\[(r, \theta) = \begin{cases} 
[x, 1], & r = |x| \text{ and } \theta = \text{Arg}(x) \\
[u, 0], & r = \infty \text{ and } \theta = \text{Arg}(u) \\
[0, 0] 
\end{cases}\]
Construction

\[(r, \theta) = \begin{cases} 
[x, 1], & r = |x| \text{ and } \theta = \text{Arg}(x) \\
[u, 0], & r = \infty \text{ and } \theta = \text{Arg}(u) \\
[0, 0], & r = \Phi \text{ and } \theta \in (-\pi, \pi] 
\end{cases} \]
Construction

\[ \mathbb{C}^T = \mathbb{C} \cup \{ (\infty, \theta); \theta \in (-\pi, \pi] \} \cup \{ \Phi \} \]
Construction

\[ C^T = \mathbb{C} \cup \{ (\infty, \theta); \theta \in (-\pi, \pi] \} \cup \{ \Phi \} \]
Construction

\[ \mathbb{C}^T = \mathbb{C} \cup \{ (\infty, \theta); \theta \in (-\pi, \pi] \} \cup \{ \Phi \} \]
Construction

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Construction

\[ \mathcal{C}^T = \mathbb{C} \cup \left\{ (\infty, \theta); \theta \in (-\pi, \pi] \right\} \cup \{ \Phi \} \]
1 ÷ 0 = [1,1] ÷ [0,1] = [1,1] × [0,1]⁻¹

= [1,1] × [1,0] = [1 × 1, 1 × 0]

= [1,0]
Examples

\[
1 \div 0 = [1,1] \div [0,1] = [1,1] \times [0,1]^{-1}
\]

\[
= [1,1] \times [1,0] = [1 \times 1, 1 \times 0]
\]

\[
= [1,0] = (\infty,0)
\]

polar
Examples

\[0 ÷ 0 = [0,1] ÷ [0,1] = [0,1] \times [0,1]^{-1}\]

\[= [0,1] \times [1,0] = [0 \times 1, 1 \times 0]\]

\[= [0,0]\]
Examples

\[ 0 \div 0 = [0,1] \div [0,1] = [0,1] \times [0,1]^{-1} \]

\[ = [0,1] \times [1,0] = [0 \times 1, 1 \times 0] \]

\[ = [0,0] = (\Phi, 0) \text{ polar} \]
Examples

\[ i \div 0 = [i,1] \div [0,1] = [i,1] \times [0,1]^{-1} \]

\[ = [i,1] \times [1,0] = [i \times 1,1 \times 0] \]

\[ = [i,0] \]
Examples

\[ i \div 0 = [i,1] \div [0,1] = [i,1] \times [0,1]^{-1} \]

\[ = [i,1] \times [1,0] = [i \times 1,1 \times 0] \]

\[ = [i,0] = \left( \infty, \frac{\pi}{2} \right) \]

polar
Value
Reach and Reliability

- Construction proves the consistency of the transcomplex and transreal numbers from the complex numbers
- Transcomplex numbers allow the solution of mathematical and physical problems at singularities
- Transcomplex numbers make mathematical software more reliable
Transcomplexes are at the beginning of their development.